Part VI

Miscellaneous Considerations

Unspecified Imbedding Space For a Point

Note a unit real single point (eg.,"1") is as mathematically simple as we can get but yet unit real and imaginary numbers N are equally simple, they just differ by the sign on the right side of $N^2=\pm 1$. Also the statement that there are "No other postulates "(other than that simple "real and imaginary" (s,θ)=point Z) at least implies there is unspecified imbedding space dimensionality and coordinate axis' position (implying measurement uncertainty ds,dθ) and possibly many more such unknowns.

Occam's Razor

We start with Occam's razor.

Thus we most *simply* either postulate a real or imaginary point or in general both (ie.,complex Z). SoWe could most simply either postulate a real or imaginary point or in general both. Furthermore we can always (in general) displace the (0,0,,..) origin by constants s_o, θ_o of arbitrary large constant magnitude with $ds \ll s_o$, $d\theta \ll \theta_o$ in $\theta=\theta_o+d\theta$, $s=s_o+ds$ giving us finally constant $Z=(s_o+ds)e^{i(\theta o+d\theta)} \equiv (r,it)$.

Part VI is optional because it mostly uses alternative approaches to obtain the same results as equation 4.2 with the rest of the sections being miscellaneous results. For example we show how to construct the section 4.3 part of the theory using Lagrangians and alternatively using the Cartan wedge product formalism to accomplish the same thing. We can also dig deeper into the CP violation section 13.1 showing the effect of CP violation on the Dirac equation and Energy dependence of CP violation and some ways to test this new interpretation of CP violation. Also the relation of this idea to supersymmetry is discussed here. Furthermore we mention the role of the Proca equation in the W and Z calculation of the Chapter 16 S matrix. These S matrix resonances must obey the Proca equation. Finally there are more details illustrating the ungauged GR theory aspect of section 12.3.

25.1 LAGRANGIAN FORMALISM

We can use section 4.3 to write a Lagrangian formalism for this theory. Again it would have both a Dirac and Einstein equation term. But of course the source is the E&M source of equation 4.2 and we have incorporated the metric coefficients $g_{\mu\nu}$ into the Dirac equation Lagrangian in equation 1.9. So on a given M th fractal scale we have from equation 1.9:

$$L_M = \left(i\psi^t \gamma_\mu \sqrt{g_{\mu\mu}}\psi_{,\mu}\right)_M + \left(m\psi^t\psi\right)_M$$

$$I = I_{Source} + I_{E\&M}$$

Our E&M Lagrangian for the free field general relativity here is $L_{E\&M} = R\sqrt{g}$

But on a general fractal scale we have from the Source Lagrangian L_{Source} the variation on the action on a given fractal scale M:

$$\delta\left(I_{Source}\right)_M = \frac{1}{2}\int d^4x\sqrt{\left(g(x)\right)_M}\left(Z^{\mu\nu}\right)_M(x)\delta\left(g_{\mu\nu}(x)\right)_M$$

Again the Lagrangian for the free field general relativity in this E&M formulation is $L_{E\&M_M} = R_M\sqrt{g_M}$ so the action is:

$$\left(I_{E\&M}\right)_M \equiv \frac{1}{16\pi k}\int\sqrt{\left(g(x)\right)_M}\left(R(x)\right)_M d^4x\,,\ k = \left(\frac{1}{4\pi\varepsilon_0 m}\right)_M$$

so the variation of the action is:

$$\delta\left(I_{E\&M}\right)_M = \frac{1}{16\pi k}\int\sqrt{g_M}\left[\left(R^{\mu\nu}\right)_M - \frac{1}{2}\left(g^{\mu\nu}\right)_M R_M\right]\delta\left(g_{\mu\nu}\right)_M d^4x$$

and so our lagrangian becomes:

$$L_{fractal} = \sum_{M=1}^{\infty}\left(i\left(\psi^t\right)_M\gamma_\mu\left(\sqrt{g_{\mu\mu}}\psi_{,\mu}\right)_M + m\left(\psi^t\right)_M\psi_M + \sqrt{g_M}R_M + \left(L_{Source}\right)_M\right) \qquad (25.1)$$

with L_{Source} defined above and

$\left(Z_{00}\right)_M \equiv \left(\frac{8\pi e^2}{mc^2}\right)_M \delta(0)$. $\left(x_\mu\right)_M$ and ψ_M and $\left(\psi_{,\mu}\right)_M$ and $\left(g_{\mu\nu}\right)_M$ are the variables used in the Noether's theorem application of the lagrangian of equation F1. Iterate the resulting Dirac equation to get the Klein Gordon equation in χ with $\frac{1}{2}\left(1\pm\gamma^5\right)\psi = \chi$.

$Z^{\mu\nu}$ is the source here.

This Lagrangian method is an alternative formulation of the theory of equation 4.2

Summary: This particular Lagrangian application appears to offer nothing new and offers us more complication and confusion instead.

25.2 Alternative Wedge Product Formulation of the Equation 4.2 Review of Second Exterior Derivative of a Vector

Here we review the second exterior derivative to show how this k-slash first derivative definition ($\gamma^i\partial/\partial x^i$ instead of $\partial/\partial x^i$) must be substituted into it. From Chapter 14 of "Gravity" (Misner, Thorne, Wheeler, MTW) define the wedge (^) product from

$\boldsymbol{u}\wedge\boldsymbol{v}=\boldsymbol{u}\otimes\boldsymbol{v}-\boldsymbol{v}\otimes\boldsymbol{u}$ where ⊗ stands for the usual tensor product. Define ds from

$ds^2 \equiv(\boldsymbol{\omega}^t)^2+-(\boldsymbol{\omega}^r)^2+(\boldsymbol{\omega}^\theta)^2+(\boldsymbol{\omega}^\phi)^2;\ (\equiv\sum g_{\mu\mu}dx^\mu dx^\mu);\ \mathbf{d}g_{\mu\nu}=\omega_{\mu\nu}+\omega_{\nu\mu}$

Where in spatial one spatial dimension $\qquad ds^2=\equiv(\boldsymbol{\omega}^t)^2-(\boldsymbol{\omega}^r)^2 \qquad$ (25.7)

given that $\boldsymbol{\omega}\ (=\sqrt{(g_{uu}dx^\mu)}$ is defined to be a one form and time 't' is imaginary.

Define $\boldsymbol{de}_\mu$ exterior derivative properties of a basis form $\boldsymbol{\omega}^\nu{}_\mu$ from

$\mathbf{de}_\mu\equiv\mathbf{e}_\nu\boldsymbol{\omega}^\nu{}_\mu;\ \mathbf{dv}\equiv\mathbf{de}_\mu v^\mu+\mathbf{e}_\mu\mathbf{d}v^\mu$; can show $\mathbf{d}^2p=0,\ p=scalar$ $\qquad$ (25.8)

Define $R_\nu{}^\mu$ and R from $\qquad R_\nu{}^\mu\equiv\mathbf{d}\boldsymbol{\omega}_\mu{}^\nu+\boldsymbol{\omega}^\mu{}_\alpha\wedge\boldsymbol{\omega}^\alpha{}_\nu;\ R\equiv\mathbf{e}_\mu\otimes\boldsymbol{\omega}_p R^\mu{}_\nu$ $\qquad$ (25.9)

The exterior derivative of a vector $\mathbf{v}=\mathbf{e}_\mu v^\mu$ is using 25.10,25.11, 25.12,25.13:

$$\mathbf{dv}=\mathbf{e}_\mu(\mathbf{d}v^\mu+\boldsymbol{\omega}^\mu{}_\nu v^\nu{}_\mu) \qquad (25.10)$$

Apply $\boldsymbol{d}$ again on 25.10 and obtain

$$\mathbf{d}^2\mathbf{v} = \mathbf{de}_\alpha\wedge(\mathbf{d}v^\alpha+\boldsymbol{\omega}^\alpha{}_\nu v^\nu)+\mathbf{e}_\mu(\mathbf{d}^2v^\mu+\mathbf{d}\boldsymbol{\omega}^\mu{}_\nu v^\mu-\boldsymbol{\omega}^\mu{}_\nu\wedge\mathbf{d}v^\nu) =$$

$$\mathbf{e}_\mu(\boldsymbol{\omega}^\mu{}_\nu\wedge\mathbf{d}v^\alpha + \boldsymbol{\omega}^\mu{}_\alpha\wedge\boldsymbol{\omega}^\alpha{}_\nu v^\nu+\mathbf{d}^2v^\mu + \mathbf{d}\boldsymbol{\omega}^\mu{}_\nu v^\nu - \boldsymbol{\omega}^\mu{}_\alpha\wedge\mathbf{d}v^\alpha) = \qquad (25.11)$$

But one has automatically for a standard exterior derivative applied twice to a scalar function $\boldsymbol{d}^2 v^\mu = 0$. So substituting the definitions 25.8 ,25.9 into 25. 11:

$$\boldsymbol{d}^2\boldsymbol{v} = \boldsymbol{e}_\mu R^\mu{}_\nu v^\mu{}_\nu \equiv \boldsymbol{Rv} \qquad (25.12)$$

$R^\mu{}_\nu$ is constructed from space-time derivatives of metric coefficients g_{ij} thus it is zero if (lets say in x, y, z, ct coordinates) space is flat so $\boldsymbol{Rv}$ is zero if space is flat. Also $Z_u{}^v$ field source from (see MTW reference):

$$R_u{}^v - (1/2)\delta_u{}^v R \equiv Z_u{}^v \qquad (25.13)$$

analogous to equation 4.2 (which comes from our equations 1.5-1.6 in any case).
Recall Cartan's boundary of a boundary is zero idea. Thus we can derive the Bianchi identities $\boldsymbol{Rdv}=0$ from equations 25.12 and 25.13 and use them to show that $Z_u{}^v$ is conserved so therefore Z_{oo} is a de Note $\gamma^j \sqrt{g_{ij}}\ \omega$ (times $-i\omega/ds$ is later called $k\!\!\!/$) in the first derivative and vector operator definitions (6) must end up in $\boldsymbol{d}^2\boldsymbol{v}$ as well. Can make use the Fourier expansion $g_{ij} = \sum e_{ij} e^{ikx}$ with $k \rightarrow k\!\!\!/$, $v \rightarrow v\!\!\!/$ to include it.

Summary: Cartan's theory contains new symbolism but the same physics as equation 4.2. (which comes out of our equations 1.4-1.6 in any case).
Also the spin in the (Cartan) torsion is classical, not quantum mechanical so cannot be substituted for the Dirac equation spin ½.Also torsion does not give the gyromagnetic ratio properties the Dirac equation gives us.

Summary of this review: Consistent with relativity we can then define ds from $ds^2 \equiv \omega_t^2 - \omega_x^2$, where $\omega_i \equiv \sqrt{g_{ii}} dx^i$ and g_{ij} is the metric coefficient. Next define P_i and k_i from $P_i \equiv k_i \equiv \omega_i/ds$. Define γ^j and χ from:$(\gamma^j P_i)^2 \chi = 1\chi$ and $k\!\!\!/$ from $-i\gamma^j \omega_i /ds \equiv k\!\!\!/$. Finally define ψ from-$i\gamma^j P_i \psi \equiv \gamma^j \partial\psi/\partial x^i$. Also we are given a density Z_{oo} and Einstein equations from the second exterior derivative $\boldsymbol{d}^2\boldsymbol{v} = \boldsymbol{e}_\mu R^\mu{}_\nu v^\mu{}_\nu \equiv \boldsymbol{Rv}$.
What is new is that we didn't drop the $\sqrt{g_{ij}}$ and $k\!\!\!/$ terms in this review.

Define CONTINUITY

A is a quantum vector operator.

Next define the covariant (so tensor) derivative: $A_{i,j} = \partial A_i/\partial x^j - [(½)m^{\alpha k}\{\partial m_{ik}/\partial y^i + \partial m_{jk}/\partial y^j - \partial m_{ij}/\partial y^k\}] A_\alpha$ And then use

$A_{i,jk} - A_{i,kj} \equiv S^\alpha_{ijk} A_\alpha$ to define **continuity** ***in the standard way*** so therefore for the case for continuity we have $S^\alpha_{ijk} = 0$. So expanding $A_{i,jk} - A_{i,kj} = [(½)m^{\alpha\chi}\{\partial m_{i\chi}/\partial y^k + \partial m_{k\chi}/\partial y^i - \partial m_{ik}/\partial y^\chi\}][(½)m^{\beta\chi}\{\partial m_{\alpha\chi}/\partial y^j + \partial m_{j\chi}/\partial y^\alpha - \partial m_{\alpha j}/\partial y^k\}]A_\beta \partial[(½)m^{\alpha\chi}\{\partial m_{i\chi}/\partial y^j + \partial m_{j\chi}/\partial y^i - \partial m_{ij}/\partial y^\chi\}]/\partial x^k A_\beta -$
$[(½)m^{\alpha k}\{\partial m_{ik}/\partial y^j + \partial m_{jk}/\partial y^i - \partial m_{ij}/\partial y^k\}][(½)m^{\beta\chi}\{\partial m_{\alpha\chi}/\partial y^k + \partial m_{k\chi}/\partial y^\alpha - \partial m_{\alpha k}/\partial y^\chi\}] A_\beta +$
$[\partial[(½)m^{\alpha\chi}\{\partial m_{i\chi}/\partial y^k + \partial m_{k\chi}/\partial y^i - \partial m_{ik}/\partial y^\chi\}]/\partial x^i\ ] A_\alpha = S^\alpha_{ijk} A_\alpha$, Use the contraction of S^α_{ijk} to define

$S_{ij} \equiv S^k_{ijk}$ and $S \equiv s^{ij} S_{ij}$. A conserved Z_{ij} is then defined from:

$$S_{ij} - (½)m_{ij} S = Z_{ij} \qquad \text{and } \sqrt{\kappa}\,\nabla\!\!\!\!/\ \psi = 0$$

25.3 What is Supersymmetry?

Recall that supersymmetry creates graded algebras connecting the boson and fermion algebraic operators (e.g., ch.17). And why shouldn't there exist such algebras trivially if couplings exist between fermion pairs, since bosons are automatically created by such couplings (e.g., singlet and triplet states)? So the existence of the coupling is really the source of the supersymmetry algebra.

Originally supersymmetry was invoked to cancel out the masses of the particles that create infinities in the standard model through supersymmetric boson partners to the fermions lets say. But here the vacuum cancellation is already done (so that the fractalness and postulated single source satisfied) the left handed neutrino is a result in section 4.5. Thus supersymmetry may be trivially satisfied here, we just haven't written down the algebra that results from there being the couplings of Fermions that creates the bosons. That information is in the couplings anyway so why bother? But there is no need here for bosonic partners to cancel infinities in the vacuum, that left handed neutrino did that. Recall $\sum_M H_M=0$ section 4.5. Setting $<\psi|Z_{oo}|\chi>\neq 0$ in equation 3.2 connects the χ bosons and fermions ψ and provides the source of the supersymmetry algebra and commutation relations.

In any case, the observability result equation 4.6 provides a natural explanation of the properties of supersymmetry. It doesn't have to be postulated out of the blue here as it is in modern versions of the standard model.

25.4 Proca equation and section 16.2

Given Equation 4.2:

$$R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R=Z_{\alpha\beta} \tag{25.5}$$

Recall that we solve these equations for this point source ($Z_{00}\equiv k\delta(\vec{x})$) and get the metric $\kappa_{\alpha\alpha}$ coefficients. Put this *spherical* source $\kappa_{\alpha\alpha}$ into the diagonalized metric equation (which could be used to define ds if we wished):

$$ds^2=\kappa_{\alpha\alpha}d(x^\alpha)^2+\ldots \tag{25.4}$$

Then define the 4 momentum[1] from (recall relativistically $p_o \propto m_o c^2 + KE$)

$$p_\alpha \equiv dx^\alpha/ds \text{ (no sum on } \alpha\text{) and define } \chi \text{ from:} \tag{25.5}$$

$$\frac{dx^\alpha}{ds}\chi=i\eta^{\alpha\beta}\frac{d\chi}{dx^\alpha} \tag{25.6}$$

Then multiply equation J4 by χ/ds^2, substitute 25.6 into 25.5 and then into J4 and get the Klein Gordon equation for χ.

$$\Box^2\chi+(1)\chi=0 \tag{25.7}$$

It can be shown that in the weak field approximation of section 12.3, while using the harmonic coordinates (the measurement point is outside matter in this example so that $Z_{oo}=0$) that equation (25.4) reduces to $R_{ij}=0$, causing the metric[1] coefficients to become:

$h'_{\mu\nu} = e'_{\mu\nu}\exp(-ik_\lambda x^\lambda) + e'_{\mu\nu}\exp(-ik_\lambda x^\lambda)$ where $\eta_{\mu\nu} + h_{\mu\nu} = g_{\mu\nu}$, $\eta_{\mu\nu}$ ≡Minkowski Metric (25.8)

Using the harmonic coordinate transformation we can write:

$$e'_{11} = e_{11} \qquad e'_{12} = e_{12} \tag{25.9}$$

$$e'_{13} = e_{13} + k\,\varepsilon_1 \qquad e'_{23} = e_{23} + k\,\varepsilon_2 \tag{25.10}$$

$$e'_{33} = e_{33} + 2k\,\varepsilon_1 \qquad e'_{oo} = e_{oo} - 2k\,\varepsilon_o \tag{25.11}$$

In general the p_i in 25.5 is a function also of a Maxwell A_μ plus a non Maxwell A_μ' (since Einstein's equations are nonlinear) so then must be χ in 25.6. We define χ by itself to be purely Maxwellian. Note we have the nonMaxwellian off diagonal contributions. Thus there is in general also a dependence on nonMaxwell equation contributions as well as the strictly Maxwell equation contributions. In the non-Maxwellian region $\chi \propto$ k and $A_\mu' \propto$k as seen in the spin 0 and spin 1 equations 25.10 and 25.11 above. There the k (and A_μ') overwhelms the ordinary linear Maxwell equation A_μ field contributions. In the weak field region there is also a huge dependence on k but derivatives are dominated by ordinary Maxwell terms. We can in general then do a Taylor expansion of χ with the respect to the field A_μ', with the first order part being the perturbed nonMaxwell part A_μ' . $\chi' = \chi + \frac{\partial\chi}{\partial A_\mu} A_\mu' + \ldots$ In the Proca formulation $\partial\chi/\partial A_\mu' \equiv (1/2)m^2$, the constancy of which we are allowed to use here because of that dependence of both χ and A_μ' on k. This perturbative A_μ' gives rise to the Proca equation $\square^2 A_\mu' + (1)A_\mu' = 0$ for a massive spin 1 particle (the W) since χ' now multiplies ($\square^2 + (1) = 0$) instead of just χ as in equation 25.6 and A_μ' contribution overwhelming the χ magnitude near the boundary. Note that the non Maxwell term above also gave the weak interaction in the previous section and that is the case here. Section 16.1 gives this W mass. Here we have just derived the wave equation iteration of equation 1.9 W mass.

25.4 Details of equation 4.11 nonzero background metric calculation using equation 4.2

For spherical symmetry from equation 4.2:

$$R_{11} = \frac{1}{2}\mu'' - \frac{1}{4}\lambda'\mu' + \frac{1}{4}(\mu')^2 - \frac{\lambda'}{r} = 0 \qquad (25.20)$$

$$R_{22} = e^{-\lambda}\left[1 + \frac{1}{2}r(\mu' - \lambda')\right] - 1 = 0 \qquad (25.21)$$

$$R_{33} = \sin^2\theta\left\{e^{-\lambda}\left[1 + \frac{1}{2}r(\mu' - \lambda')\right] - 1\right\} = 0 \qquad (25.22)$$

$$R_{00} = e^{\mu-\lambda}\left[-\frac{1}{2}\mu'' + \frac{1}{4}\lambda'\mu' - \frac{1}{4}(\mu')^2 - \frac{\mu'}{r}\right] = 0 \qquad (25.23)$$

$$R_{ij} = 0 \quad \text{if } i \neq j$$

Equation 25.22 is a mere repetition of equation 25.21. We thus have only three equations on λ and μ to consider. From equations 25.20, 25.23 we deduce that $\lambda' = -\mu'$ so that $\lambda = -\mu +$ **constant** $= -\mu + C$. So $e^{-\mu+C} = e^{\lambda}$. Then 24.21 can be written as:

$$e^{-C}e^{\mu}(1 + r\mu') = 1 \qquad (25.24)$$

Set $e^{\mu} = \gamma$. So $e^{-\lambda} = \gamma e^{-C}$ and so integrating this first order equation (equation.3.24) we get:

$$\gamma = -\frac{2m}{r} + e^C \equiv e^{\mu} \text{ and } e^{-\lambda} = \left(-\frac{2m}{r} + e^C\right)e^{-C} \qquad (25.25)$$

Now we substitute 25.25 into the metric of equation 1.1. The result is:

$$ds^2 = \frac{(dr)^2}{1 - \frac{2m'}{r}} - r^2(d\theta')^2 - r^2\sin^2\theta\ '(d\phi')^2 + \left(1 - \frac{2m}{r} + \left[e^C - 1\right]\right)c^2(dt')^2. \qquad (25.26)$$

25.6 Continuation of chapter 2: Summary of Fractal implications

Review

Historically, a point assumption of the type $\delta\int ds = 0 \equiv \delta ds = 0$ gives a lot of physics (geodesics) but requires extra (field) equation results in the ds formulation. But a point must also be able to move at a constant angle $d\theta$. The string Nambu action minimizes an area $A = ds(s_o d\theta)$ lets say and so makes some headway in minimizing $\delta d\theta$ but to really get at what a point is you must minimize $\delta d\theta$ independently. That is what is new about this idea. Applying $\delta ds = 0$ and $\delta d\theta = 0$ independently then allows you get everything, including the field equations.

In that regard the **assumption of a geometrical point** is the simplest possible starting point of theoretical physics. But for it to be a point and not a mere line connecting to the origin it must also be a *point* rate of motion and so we require the complex plane so $Z = (s_o + ds)e^{i(\theta o + d\theta)}$ (giving a point motion world line). Furthermore if it is *necessarily a point* it can move only in a single direction (so $\delta d\theta = 0$) for single distance s_o ($\delta ds = 0$) and all other coordinate systems (such as dr_1 and dt_1 in $dZ=dr_1+idt_1$) have to conform to this requirement. So:

postulate point $Z=(s_o+ds)e^{i(\theta o+d\theta)}$ 1

for constant ds and dθ.

Postulate geometrical point: $Z=(s_o+ds)^{i(\theta o+d\theta)}$ 1.1

Constant ds and dθ

Geometrical point exhibits no variation ds and dθ $\delta ds=0$, $\delta d\theta=0$ 1.2

Define dZ≡dr+idt. So $\delta d\theta=\sin\theta dr/so+\cos\theta dt/so=(1/(so\sqrt{2}))(dr+dt)=0$ resulting in $\delta(dr+dt)=0$ and so dr+dt=constant =dr-ε/2+dt+ε/2=dr'+dt". Eq. 1.3 with ε higher order terms in the series 1.2.

This appears to give all the physics without either implicitly or explicitly adding any more postulates or assumptions!

Starting with δ(dr+dt)=0 zero out each term added to dZ. (WE GET A LOT OF EQUATIONS!) For example the Triangle inequality makes δ(ds)=0 (ds=|dZ|) Then the $ds^2=dr^2+dt^2$ makes ε added to dr small (|r|<ε<<dr)so the series converges rapidly.

dr' and dt' included in dr+dt=constant give that new (Dirac) pde in dZ with its r_H.

Cross terms in 1.2 give zero resulting in the left handed Dirac doublet. Each N th term in the eq.1.2 series acts expansion as a separate fractal scale with that r_H boundary between them. Gravity (section 12.1) results from the effect of the expansion of the previous scale (in eq. 1.2 expansion) and on the given next smaller scale. Thus even the fractalness, gravity and the left handed doublet are explicit in eq.1.2! Note I have not added any new assumptions to that eq. 1.1 postulate!! Just follow where it leads.

My fractal selfsimilar objects of equation 1.2 are the 10^{10} light year diameter "universe" and the electron. The universe expands and the zitterbewegung oscillation of the electron implies that it also has an expansion stage. The electron and the universe are essentially then the same structures but viewed from radically different scales. If you were 10^{40} times larger (10 with 40 zeros) the universe we are in would once again appear to you as a mere electron.

Essentially, then, astronomers are studying from the inside (CBR, galaxy clusters, etc.) what particle physicists are studying from the outside (electrons). On some starry night contemplate that as you look up at sky! When this fractalness is formulated in the context of that UNgauged GR there is a tremendous simplification of the physics (i.e., it has the ring of truth to it). In that regard:

This paper can then be formulated alternatively as an UNgauged general relativity. Ordinary general relativity has 6 independent equations (due to the Bianchi identities) and 10 unknowns, which is a algebraicly INcomplete system. The usual procedure to make it complete is to add 4 arbitrary equations. Here we will show that the choice of equations *isn't arbitrary* and simply acknowledging this simple fact results in straightening out all the difficulties that are plaguing theoretical physics lately.

25.7 Continuation of section 12.3: Algebraic Completeness and Gauges Algebraic (grade school example tutorial)

3 equations and **3 unknowns**:
2=x+2y+3z
2=3x+1y+2z
1=2x+3y+1z
solution x=.2777.., y=-.0555..,z=.06111..
Also note the same number of unknowns as equations.
But what if some theory (lets say that we knew that it was valid) gave us only:
2=x+2y+3z
2=3x+1y+2z

But then what if I did not know of a third equation AND SO I SIMPLY MADE UP out of the blue
1=2x+3y+1z ????????
Then I said that my theory implied that x=.2777…, y=-.0555..., z=.06111…
Making up that extra equation is the essence of gauge theory! What a sad direction physics is taking.

25.7 Algebraic Completeness

Recall equation 4.2:

$(R_{ij}-\frac{1}{2}g_{ij}R)A|a,t\rangle = Z_{ij}A|a,t\rangle =$ or 10 equations since $i \rightarrow 0$ to 3 and $Z_{ij}=Z_{ji}$ (25.1)

We drop for now writing the $A|a,t\rangle$ but note that it is assumed below in all equations. In any case, the Bianchi identities ($Z^{\mu}_{\nu;\mu}=0$) make this only **6 algebraicly independent equations**.

Note that the unknowns are g_{11}, g_{12}, g_{13}, g_{10}, g_{21}, g_{22}, g_{23}, g_{20}, g_{31}, g_{32}, g_{33}, g_{30}, g_{01}, g_{02}, g_{03}, g_{00} but due to g_{ij} metric symmetry in ij we have $g_{21}=g_{12}$, $g_{31}=g_{13}$, $g_{01}=g_{10}$, and therefore only **10 unknowns.**

These equations in the free space weak field approximation are for example (with $g_{ij}=1-h_{ij}$: with $|h_{ij}|<<1$) :

$$R_{\mu\mu} \approx \Box^2 h_{\mu\nu} - \frac{\partial^2}{\partial x^{\lambda}\partial x^{\mu}}h^{\lambda}_{\nu} - \frac{\partial^2}{\partial x^{\lambda}\partial x^{\nu}}h^{\lambda}_{\mu} + \frac{\partial^2}{\partial x^{\mu}\partial x^{\nu}}h^{\lambda}_{\lambda} = 0 . \quad (25.2)$$

(in the Fourier decomposition of a diagonal $\mu=\nu$ metric $g_{\mu\nu}=\sum a_k x^{ikx}$ the above equation is rewritten as $R_{\mu\nu} \approx k^{\alpha}k_{\alpha}h_{\nu\nu}-k_{\nu}(2k_{\mu}h_{\nu}^{\mu}-k_{\nu}h_{\alpha}^{\alpha})=0$). Note here and in equation 4.2 that we have not yet conjugated out the zitterbewegung oscillation. For $\mu=\nu$ we substitute the four harmonic coordinate gauge conditions:

$$\Box^2 h_{\mu\nu}=0 \text{ (or } k^{\alpha}k_{\alpha}h_{\mu\nu}=0 \text{ in the Fourier expansion of } h_{\mu\nu}) \quad (25.3)$$

and obtain from equation 4.2:

$$-\frac{\partial^2}{\partial x^{\lambda}\partial x^{\mu}}h^{\lambda}_{\nu} - \frac{\partial^2}{\partial x^{\lambda}\partial x^{\nu}}h^{\lambda}_{\mu} + \frac{\partial^2}{\partial x^{\mu}\partial x^{\nu}}h^{\lambda}_{\lambda} = 0$$

Implying the harmonic coordinate gauge equations after integrating:

$$\frac{\partial}{\partial x^{\mu}}h^{\mu}_{\nu} = \frac{1}{2}\frac{\partial}{\partial x^{\nu}}h^{\mu}_{\mu} \text{ ,or in the above Fourier expansion k, } k_{\mu}e^{\mu}_{\nu} = \frac{1}{2}k_{\nu}e^{\mu}_{\mu} \quad (25.4)$$

Thus **equation 25.4 is a gauge condition** with other coordinate gauge conditions *also* possible. The mathematical outcome then is somewhat *arbitrary*.

The usual method to remedy this problem is to convert general relativity into a gauge theory. For example these four harmonic gauge equations provide the needed additional equations making general relativity complete.

However, solving the Heisenberg equations of motion for a Dirac equation (i.e, $\sqrt{g_{ij}}\nabla\psi+i\kappa\psi=0$) Hamiltonian gives a $r \propto \exp(i\omega t)$ oscillation and therefore an equation 25.3 oscillation. Thus the (ubiquitous) Dirac particle already exists in a *real* physical harmonic coordinate system (the zitterbewegung oscillation) so that a gauged theory is not required and consequently general relativity is actually *complete anyway*. However augmenting the Einstein equations with the Dirac equation causes the Einstein equations to become the Maxwell equations (E&M) in the weak field approximation.. In that regard we have $k_{\mu}e^{\mu}_{\nu}=\frac{1}{2}k_{\nu}e^{\mu}_{\mu}$ from equation 25.4 and $k^{\mu}k_{\mu}=0$ from equation 1.3.

Then we sum equation 25.4 (in the form $k_\mu e^\mu_\nu = \frac{1}{2} k_\nu\ e^\mu_\mu$) to obtain (with $g^{\kappa\mu}\ e_{\nu\kappa} \equiv e^\mu_\nu$) for $\mu=\nu$:

$$\frac{1}{2}\sum_\mu k_\mu\left(e_{11}+e_{22}+e_{33}-e_{00}\right)=k^\mu e'_{\mu\mu}=k^1e_{11}+k^2e_{22}+k^3e_{33}-k^0e_{00}=k^\nu e_{\nu\nu} \qquad (25.5)$$

The Dirac equation for $\sum k_i = \sum k\!\!\!/_i = 0$ comprises one set of solutions of the equation 1.3 term. $k^\mu k_\mu = 0$ (analogous to linearizing the Klein Gordon equation to obtain the Dirac equation). Thus these $k \equiv k\!\!\!/$ s can be substituted into the left side of equation 4.2 to make it zero because $\sum_{\mu=0}^{3} k_\mu = \sum_{\mu=0}^{3} k\!\!\!/_i = 0$ for the Dirac equation.

Thus $k^1e_{11}+k^2e_{22}+k^3e_{33}+k^0e_{00}=0$ holds in equation 25.5 and therefore with equations 25.3 (for $\mu=\nu$) we finally have:

$$k^\mu e_{\mu\mu}=0,\ k_\mu k^\mu e_{\nu\nu}=0 \qquad (25.6)$$

These last two equations are equivalent to **Maxwell's** equations (after taking expectation values (,<ψ|α|ψ> of the Dirac matrices or even using $e_{\mu\mu} \rightarrow e_{\mu\mu}/\gamma_\mu$, $J_{\mu\mu} \rightarrow J_{\mu\mu}/\gamma_\mu$) in the Lorentz gauge for the free space case (identical with equations 17.1, 17.2). Note after the bilinear expectation value is taken the zitterbewegung is conjugated out and the zitterbewegung cloud takes on its random character. But note that what is called the Lorentz gauge is no longer a "gauge" in this formulation. Therefore given this Maxwell equation result we can substitute in an E&M ***(8πe²/mc²) =Z***$_{oo}$

Finally note with **the four** (iteration) Dirac equation harmonic coordinate equations $\square^2 h_{11}=0$, $\square^2 h_{22}=0$, $\square^2 h_{33}=0$, $\square^2 h_{oo}=0$ (i.e., equation 25.6) **there are then 10 equations again.** Therefore the solution has
10 g_{ij} s , **6** algebraicly independent Einstein equations and **4 NON**gauge *physical* harmonic coordinate equations. Thus there are **10 equations** (plus the Dirac equation), **10 unknowns** (plus the Dirac ψ) and **no gauge condition** anymore, an **Un**gauged General Relativity

25.8 Perturbation Due to Point Source in Nonzero Ambient Metric at Infinity: $g_{oo} \propto 1+k_H/r + C$, $g_{oo} \neq 1$ as $r\rightarrow\infty$, or alternatively $\lambda=-\mu+$*constant* in equation 4.2

Recall section 4.4. Metric component asymmetry due to background terms ε and Δε. Combining equation 1.5, 1.7 ($e^\mu=\kappa_{oo}=1/\kappa_{rr}=1/e^\lambda$) and equation 4.2 ($R_{22}=0$) gives us the complete Einstein equation results. Thus there is a fractal selfsimilarity of the M+1 cosmological background metric component with the M th fractal scale Dirac equation 1.9 with it's zitterbewegung oscillation and *spin*. For this necessarily comoving observer this adds a selfsimilar oscillatory ε to a small (due to inertial frame dragging) $\Delta\varepsilon_\phi$ spin object so that $\kappa_{oo} = 1+\varepsilon+\Delta\varepsilon$ where the κ_{oo} and κ_{rr} are related by equation 1.7. Thus this is also a perturbation due to a nonzero ambient metric at infinity: $\kappa_{oo} \propto 1+k_H/r+\varepsilon$, $\kappa_{oo} \neq 1$ as $r\rightarrow\infty$, If we use the ansatz $e^\mu=\kappa_{oo}$ and $e^\lambda=\kappa_{rr}$ then equation 4.2 implies $\mu=\lambda+C$ and thus $e^{-\mu+C}=e^\lambda$. and thus equation 25.21 is written (see equation 25.21):

$$e^{-C}e^{\mu}(1+r\mu')=1$$

Set $e^{\mu}=\xi$. So $e^{-\lambda}=\gamma e^{-C}$ and so integrating this first order equation we get:

$$\xi=-\frac{2m}{r}+e^{C}\equiv e^{\mu}\text{ and }e^{-\lambda}=\left(-\frac{2m}{r}+e^{C}\right)e^{-C} \quad (25.7)$$

with again $e^C-1=\varepsilon$ or $e^C=1+\varepsilon$. Thus in the equation 4.2 metric we can take into account this new asymmetry in g_{oo} and g_{oo} provided by this new term $e^C=1+\varepsilon$ by resetting the proper time ds by multiplying both sides by $1\pm\varepsilon$ to find:

$$ds^2=\frac{(dr)^2}{1-\frac{2m'}{r}}-r^2(d\theta')^2-r^2\sin^2\theta\ '(d\phi')^2+\left(1-\frac{2m}{r}+\left[e^C-1\right]\right)c^2(dt')^2. \quad (25.8)$$

Thus to immerse the source in the nontrivial ambient metric of equation 4.2 you must somehow make the ambient metric of equation 25.8 consistent with the symmetric metric of equation 1.10. The simplest invariant way to get equation 25.8 from equation 1.6 at the $r\approx k$ horizon is to rescale the equation 1.10 ds (the effect of which is simply to rescale time keeping for the comoving observer) in which case we simply multiply the metric of equation 4.6 by $e^{C/2}\approx 1+2\varepsilon$ to get equation D3 in terms of ε (the goal being to cancel the ε in the dr^2 term). This multiplication also gives: $\theta\rightarrow e^{C/4}\theta\equiv\theta'$, $\phi\rightarrow e^{C/4}\phi\equiv\phi'$, with nonzero C/4 implying that the

$$ds^2=\frac{(1\pm\varepsilon+..)}{\left(1-(2k_s/r)\pm\varepsilon\pm\Delta\varepsilon\right)}(dr)^2+r^2(d\theta)^2+r^2\sin^2\theta(d\phi)^2-(1\pm\varepsilon+..)\left(1-(2k_o/r)\pm\varepsilon\pm\Delta\varepsilon\right)(dt)^2$$

where $k_o\equiv k_H/(1\pm\varepsilon)$, $k_H\equiv\langle Z_{oo}/4\pi\rangle$. For S states we note that $m_\tau=2m_p/(1+\varepsilon)$ in $Z_{oo.}$ This is the more expanded proof of equation 4.5 in chapter 2.

So, to be able to write the Dirac equation in the standard way, the ds^2 in the geodesic equations must then be multiplied through by $1\pm\varepsilon$. For neutral particles, there is only the $1-\varepsilon$, and for the electron the $1+\varepsilon$ in the denominator cancels out, and at those low energies $\varepsilon=0$ in front of the dr^2, leaving only $1+\Delta\varepsilon$. ε is imaginary inside $r<r_H$ if it is real outside, so it does not then cancel the ε term in 1.10. Thus, in the Frobenius solution to equation 1.9, ε remains inside the g_{22} metric term.

25.9 Alternative (Fractal) Calculus: New Derivatives and Integrals

We can also state our equation theory as an alternative form of calculus with new forms of derivatives::

Derivative of ψ given by equation 1.9: $\sqrt{\kappa_{\mu\mu}}\gamma^{\mu}\partial\psi/\partial x^{\mu}-\omega\psi=0$

Antiderivative of ψ given by equation 3.2: $\int\psi k_o c\chi dv=P$ (where $\chi\equiv\frac{1}{2}(1\pm\gamma^5)\psi$).

To learn more about this new integrals and derivatives alternative approach see:
http://bellsouthpwp2.net/m/a/maker3/
To get equation editor you must Google these two sites:
http://74.125.47.132/search?q=cache:j56WUX2nGowJ:absimage.aps.org/image/MWS_MAR09-2008-000037.pdf+davidmaker&cd=28&hl=en&ct=clnk&gl=us

25.10 Implications of Our Single Postulate Postulate 1.1

Here we introduce alternative ways of stating the results of equation 1.1 that might be more intuitive. Thus equation 1.1 also implies a

1) Complex plane dZ= dr +idt with dZ not divergent (or rotational).
2) Also equation 1.1 postulate implies a **ONE spatial dimensional theory** r with time dt as the imaginary component (with then zero divergence (and rotation0 in the complex plane). Recall from chapter 1 also:
3) "**Physics is the process for observing 2D**".(since the dZ=dr+idt not divergent). Recall the QM requirement for observer and object gives a direct sum 2⊕2=4 degrees of freedom physics and a resulting new Dirac equation with horizon r_H in its κ_{oo}.New Dirac equation (sum on μ)
   $\sqrt{\kappa_{\mu\mu}}\gamma^{\mu}\partial\psi/\partial x^{\mu}-\omega\psi=0$ (1.9)
   Thus we might make the alternative, and possibly more intuitive, postulate (alternative to equation 1.1) of:
4) **ONE nonzero rest mass m_e** (in $r_H=2e^2/m_ec^2\equiv Z_{oo}/4\pi$) obeying this equation. Or we have from 1.1 the
5) **Simplest Observable Source** Z_{oo} in $r_H=Z_{oo}/4\pi$: a point source (section 4.6)

25.11 Horizons Imply Fractal Universe If Only One Type Of Source Inside Horizon

In the next section we use these alternative results and the rules of logic (instead of that Taylor expansion of equation 1.2) to prove the fractalness and so most of the consequences. Again, postulate 1.1 is still the core idea here. But it is interesting nonetheless that the existence of a event horizon naturally implies a fractal universe.

Symbolic logic application of postulated "simplest observable source" in the form of equation 4.8

The sequencing resulting from the fractal (scale sequence) implies mathematical induction. That result has consequences in the development of the theory of numbers and completeness.

Symbols

obscale = observers scale

observable =From section 4.4:No event horizon separates the observer and the observed system and

$<\sum\psi|Z_{oo}|\chi>\neq 0$

$simplest \equiv \delta(0)Z_{oo}$ (of equation 4.3)

$\exists_x\equiv$ There is some x

$\forall_x \equiv$ For all x

$\wedge \equiv$ and=conjunction

$\rightarrow$=conditional, if-then

$S_R \equiv$ Source associated with event horizon R.

Symbolic Logic Rules Used Here

$a \rightarrow b$
a
$\therefore b$ *modus Tolens*

$\forall_x (A_x \wedge B_x)$
A_x
Rule 1
$\forall_x (A_x \wedge B_x)$
B_x
Rule 2

Given two scales event horizons R_1 and R_2 of section 2.3 where FS is the scale difference between the two scales. Then scale R_1 and scale R_2 are selfsimilar if every object on scale R_1 is scaled up by the same factor FS (one scale is just a magnified version of the other or $R_2 = (FS)R_1$) of section 2.3.
Also recall section 4.4 allowing us to postulate that the Scale of observer =obscale can be any size.
So we can alternatively write down postulate 4.6 by saying (as in alternative 2 in above section 25.15)

1) $\exists_y (obscale = y) \rightarrow \exists_R (observable) \wedge (S_R \equiv simplest) \wedge (obscale > R)$
ie., **There exists the simplest possible observable source. (equation 4.8)**

Next we note the postulate 1 (or equation 4.8) implies that the Scale of observer =obscale can be any size.
2) $\forall_X (\exists_y (obscale = y) \wedge (obscale < X))$
ie., Observers can be anywhere (either outside or inside) relative to a horizon.

So $\exists_y (obscaleM = y)$ (from postulate 1 and rule 1) (25.10)
Substitute the names $obscaleM$ and R_M and S_{R_M} for obscale and R and S respectively.
Next substitute the S_M, R_M and obscaleM into postulate 1:
$\exists_y (obscale = y) \rightarrow \exists_R (observableM) \wedge (S_{R_M} \equiv simplest) \wedge (obscaleM > R_M)$ (25.11)
From modus tolens and 25.10 and 25.11 then
$\exists_R (observableM) \wedge (S_{R_M} \equiv simplest) \wedge (obscaleM > R_M)$
Rule 2 says $\forall_X$ i.e., for all X., So we take $X=R_M$. Then from rules 1 and 2
$\exists_y (obscale(M-1) = y) \wedge (obscale(M-1) < R_M = X)$ (25.12)
Rule 1 now states the existence of some y in postulate 1 that is equal to obscale(M-1). But we can write *any* obscale since this is a mere conditional statement. So we substitute obscale(M-1) in rule 1 since this is a new scale (since now obscale $<R_M$):

$\exists_y \left(obscale(M-1) = y \right) \rightarrow \exists_R \left(observable(M-1) \right) \wedge \left(S_{R_{M-1}} \equiv simplest \right)$

$\wedge \left(obscale(M-1) > R_{M-1} \right)$ (25.13)

So from 25.12 and rule 1

$\exists_y \left(obscale(M-1) = y \right)$ (25.14)

So by modus tolens using 25.13;

$\exists_R \left(observable(M-1) \right) \wedge \left(S_{R_{M-1}} \equiv simplest \right) \wedge \left(obscale(M-1) > R_{M-1} \right)$

So putting all of the inequalities in 25.11, 25.12 and 25.13 together:

$obscaleM > R_M > obscale(M-1) > R_{M-1}$.

So: $R_M > R_{M-1}$ or adding 1 to M for convenience:

$R_{M+1} > R_M$

or in general:

$\exists_R \left(observableM \right) \wedge \left(S_{R_M} \equiv simplest \right) \wedge \left(obscaleM > R_M \right)$ (25.15)

$\exists_R \left(observable(M+1) \right) \wedge \left(S_{R_{M+1}} \equiv simplest \right) \wedge \left(obscale(M+1) > R_{M+1} \right)$ (25.16)

with $R_M < R_{M+1}$ (25.17)

Combining the inequalities 25.17 for result 2 thus written in general we have that: $\ldots R_{M-1} < R_M < R_{M+1} < \ldots$ This set of inequalities in the R *s,* plus the associated existence statements such as 25.15 and 25.16, implies selfsimilarity and fractalness of the horizons R_M .

Thus our corollary to these two postulates is the selfsimilarity fractalness given scales given by existence statements 25.15 and 25.16 and ordered by:

$\ldots R_{M-1} < R_M < R_{M+1} < \ldots$ (25.18)

The point of all this was simply to show that the fractal N th scale set of integers can also then create a one to one mapping of the fractal scales and the integers. Thus mathematical induction is now trivially incorporated into the mathematics of the postulate 1.1 using this alternative symbolic logic application of the postulate 4.8.

25.12 Postulate point with second reference point allowed. Define the number 3 as 3 points, eventually using this sequence of definitions to map one to one to the natural numbers replicating the result of Peano's (inductive) axioms.

Here we postulate a real point. But we need to rigorously state what we mean by a real point. Note a real point designation(definition) must include the origin. Define a 'unit set' to be this point and the original point. So we can then randomly generate any other real number by 'adding' these origin real points (with 'addition' of 1+1=2 defined from the union of the unit sets: 1∪1≡2 of origin point and designated point to be "2 points") thereafter allowing a mathematical induction creation of the real numbers mirroring the Peano procedure (see section 25.12). Then define multiplication nXm as the set created by adding n, m times n∪n∪...m times≡nXm.. And subtraction of 1 can be defined by adding one fewer origin unit set. So we are defining the standard mathematical operations by *applying names to different types of unions of these unit (real point) sets.* Continuing on in this set theoretic manner we can derive rings and fields and thereby our standard

algebra and calculus. For example integrals are just a fancy way of doing addition. We can then define integrals of integral sets and the reverse process would define differentiation and thereby "division" of one number by another.
Given the properties of multiplication we have just defined you can then expand our definition of a real number (in our above real point postulate) to say that for a real number R that R^2= *not* negative. So that a real number is not imaginary. If you then state that there is only one real number (line) and allow for other dimensions the next dimension by default must then be imaginary C^2=negative, can't be real. Thus in general you have complex numbers Z coming out of postulating a single real number if you allow for a UNspecified imbedding dimensionality: in other words by saying there are no new postulates. Furthermore an unspecified imbedding dimensionality implies inexact small real number measurement uncertainty so $|dr|>0$ and therefore uncertainty $ds,d\theta$ in the complex plane (so $dZ=dse^{id\theta}$) and therefore a possibly small curve in the line from the origin to the point.
We could most simply either postulate a real or imaginary point or in general both.
Postulate a constant **point** so $\delta Z=0$. Because we have *no other postulates* we have an Unspecified number of imbedding dimensions and inexact position of the imbedding coordinate axis'
We **postulated a point** (ordered pair) object all the while allowing for there to be other such points, other ordered pairs. Thus we can then assume another such point defined then to be 2 objects, temporarily leaving out the minimal position variation discussion in this first step.
We could continue and with another such point so define an additional point to be 3 (third point). In that way we could construct N points. (Or we could define 3 as being 2+1 thereby defining addition, etc. could also define the multiplication of 2 and 3 as adding groups of 2 three times). In this way we can pull out 1 number and N and N+1 numbers out the long sequence we constructed and thereby get Peano's result (the natural numbers) for free without using his axioms or his induction. We have then merely a sequence of definitions where the starting point was that postulate of a point. Thus they are no longer axioms, being derived from mere definitions, with the only postulate being then that of that point with allowance for other points.
In that regard there are historically two alternatives used for defining the natural number system, one being Peano's axioms (use mathematical induction: assume 1 and N therefore N+1 integers of arbitrary large value) and the other being the laws of algebra and fields (e.g., associative, distributive, commutative laws). But again our above sequence of definitions imply there is a 1 and an N+1 number so these are not axioms anymore given that one to one mapping, they are proven. All 5 of Peano's "axioms" are implied by this sequence of definitions (these chapter 25 inductive implications) and so they are not 'axioms' anymore.
Note that (Kurt) Godel's famous incompleteness theorem uses the algebraic definitions of the integers in its proof, not Peano's "axioms".
Thus by proving Peano's axioms (so they are not axioms anymore) we detour around the algebraic definitions of the natural numbers and therefore falsify Godel's incompleteness.
Alternatively, we avoid making another postulate by just defining the properties of the real numbers and apply them to the individual components of $dZ=dr+idt$.

Thus our postulate gives us not only the physics but the underlying mathematics. We have a unified everything, both math and physics, theory

25.13 Inverse Separation of Variables of Chapter 3

Recall eq.1.9 $\sqrt{\kappa\gamma}\nabla\psi + i\kappa\psi = 0$

we have a infinite succession of such equations:

$$\ldots, (\nabla\psi + i\kappa\psi = 0)_{M-1}, (\nabla\psi + i\kappa\psi = 0)_{M}, (\nabla\psi + i\kappa\psi = 0)_{M+1}, \ldots$$

one for each fractal scale with $\kappa \equiv \omega/c$. Note from the lagrangian of equation 1 (with the Einstein equation component) the physical regions in which each of these equations apply are separated by a event horizon. The physical effects *on* the *ambient* metric vacuum begin with the Mth scale, (here being the electron scale $\sim 10^{-18}$ m lets say). Also this sequence of Dirac equations is equivalent to a single *separable* differential equation in the ψ_M s with the 1/c serving as the separation constant. Thus we can write a product function of the ambient ψ_M s:

$$\prod_{N=M}^{\infty} \Psi_N = \Psi_M \bullet \Psi_{M+1} \bullet \ldots = \Psi_{Physical}$$

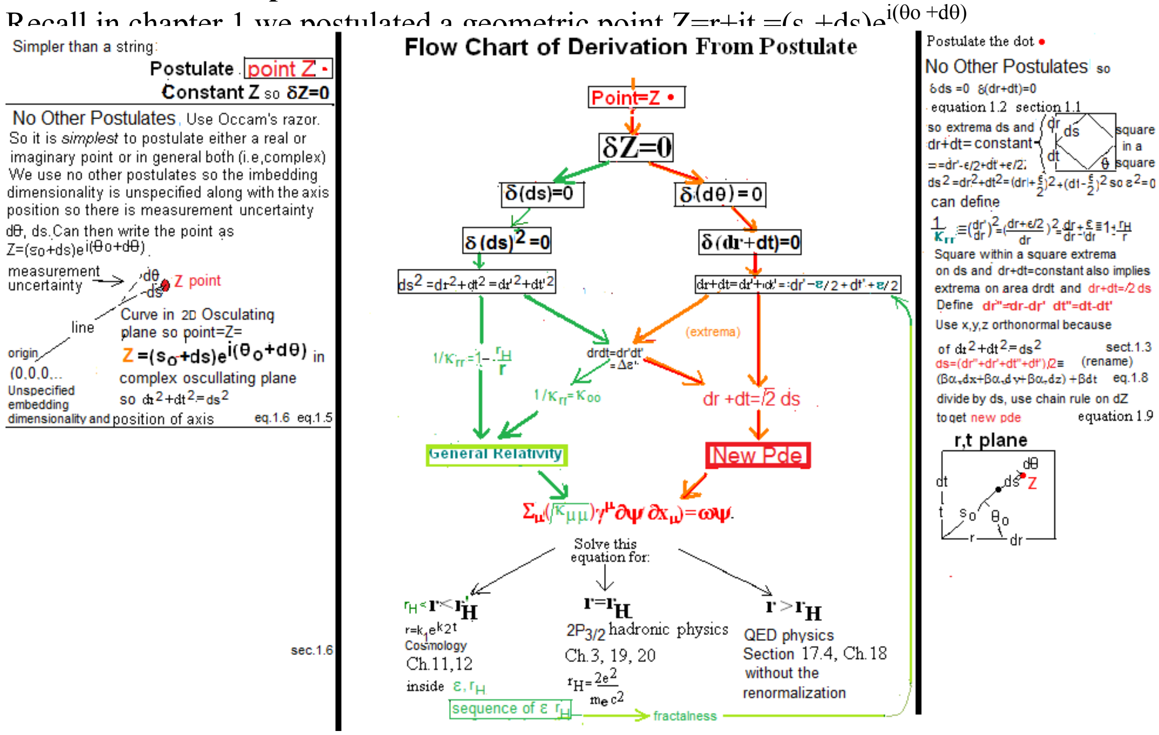
Because these Dirac eigenfunctions have the energies in their exponents ($\Psi \propto e^{i\omega t} = e^{i\langle H\rangle t/\hbar}$ with $H\psi = E\psi$) we can also write (with k a column matrix, 't' the M+1 scale proper time):

$$\Psi_{physical} = k \exp\left(i(1/\hbar)\sum_{N=M}^{\infty} H_N t \right) = k \exp\left(i(1/\hbar) H_{physical} t\right)$$

Additionally the zitterbewegung oscillation will have this same $e^{i\omega t}$ dependence (as in $r = r_o e^{i\omega t}$) from the Heisenberg equations of motion. From $dt/ds = 1/g_{oo}$ [2] and appendix equation C2 $E \propto dt/ds\sqrt{g_{oo}}$, we have $H \propto 1/\sqrt{g_{oo}}$. Thus:

25.18 Notes on Chapter 1

Recall in chapter 1 we postulated a geometric point Z=r+it =(s$_o$+ds)e$^{i(\theta o+d\theta)}$



Also recall the geometric point implied *minimal position variation* using the collapsed triangle inequality δ|dZ|≤ δ|dr|+δ|dt|=0.

This implies dr+dt=constant and δ(ds)=0 implying a square within a square geometry where we find that the extrema occurs when θ=45°, dt=dr in the diagram below since here ds is at a minima then. Note also drdt/2 is at maxima as well at θ=45°:

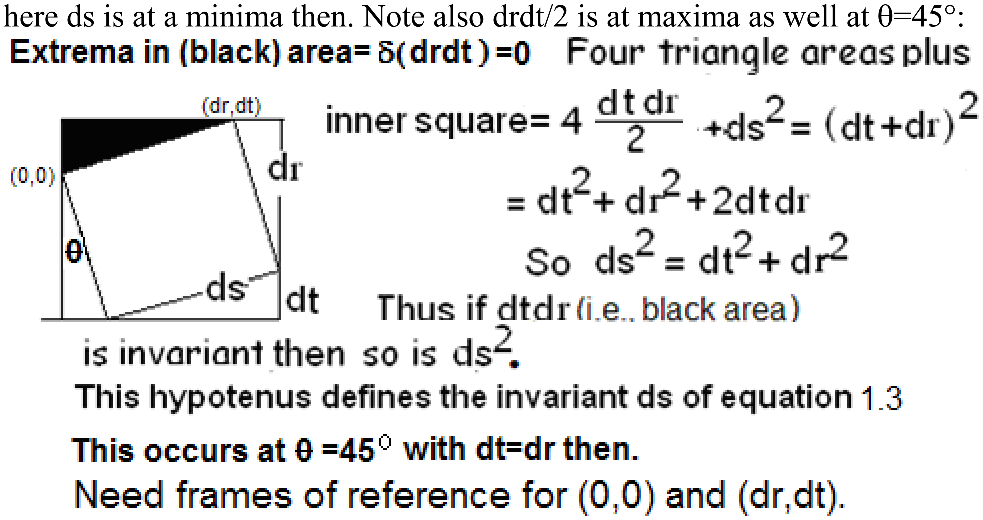


Figure 25-1

Note ds is minimized at θ=45° and also then dr=dt.

This Is A Children's Geometry Problem: Requires Only A Piece of Paper, Pencil

Postulate 1.1 is in a two dimensional space but equation 1.9 is 2D+2D=four dimensional with constraint equations 1.5 and 1.6 making the effective number of dimensions still 2.

Thus the mathematics is still that of a two dimensional space!!!! Thus this is children's geometry problem that can be done on a piece of paper!! As in figure 25-1 applied to equations 1.2a, 1.3. Just do not 'smudge the points' or diverge a geometrical point as in definition equation.1.2.

Note here that the figure 25-1 analysis of the extrema on the square within a square gives equation 1.9 and so all of the physics of our universe. But this is just a simple straight line 2D geometry problem. Thus *the most fundamental theoretical physics is child's play geometry* drawn (in crayons perhaps) *on a piece of notebook paper*!

25.19 Why Have Frames of Reference?
Postulate geometric point dZ=dr+idt

Definition of point is ordered pair (dr,dt):

From above triangle geometry geometrical distance of point from (0,0) is $ds = \sqrt{(dr-0)^2 + (dt-0)^2}$. Given that (0,0) and/or (dr,dt) points can be in motion there are frames of reference (dr',dt') for the (0,0) origin and (dr,dt) for the (dr,dt) point.

25.20 Are There Other Ways of Realizing Equation 1.3 ds Extrema δ(ds)=0 ?
Recall minimum ds (at nontrivial solution 45° result) is on result of equation 1.3.

The standard minimal geodesic path length extrema $\delta\int_{t_1}^{t_2} ds = 0$ for constant t_1 and t_2 occurs also at the minima extrema in ds at θ=45° making it a special case of the more general form (i.e, that square in a square) we are using. Infinitesimal Lie group generators (e.g., SU(2)) also keep |ds| a constant for z axis rotation SO(3) spinor representations of dZ (i.e., $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}dr \\ dt\end{pmatrix} = \begin{pmatrix}dr' \\ dt'\end{pmatrix}$ where θ<<1.

Recall from section 1.1 that at the 45° extrema that dr≈dt so that -sinθdt≡-ε/2 and sinθdr≡ε/2 establishing exact correspondence with the section 1.1 dr',dt' formulation. This result proves the uniqueness of our dt'=dr-ε/2 frame of reference choice of chapter 1 establishing the uniqueness of equation 1.9.

Note these Lie groups already form the basis of that alternative 2 (r_H=0) gauge group structure (of chapter 7) discussed at the beginning of this book which, as we saw is a mistaken approach since it assumes at the beginning that r_H=0. SU(2) is implicit in the structure of the Dirac matrices in equation 1.9 already and so otherwise these Lie groups can be ignored.

Hamilton's principle extrema on L=T-V (as in δ∫Ldt=0) allows us to organize our dynamical equations and state Noether's theorem (so has some utility in classical mechanics) but actually *adds* extra postulates in practice in more fundamental applications such as those of chapter 1. For example in the standard Lagrangian formulations we need a Lagrangian for the Dirac equation, a Lagrangian for E&M, a Lagrangian for mechanics, etc.plus all the q and qdot assumptions which amounts to many new assumptions thereby confusing theoretical physics in the final analysis. In any case the results we get come directly out of our new pde (eq.1.9) without any recourse to these many Lagrangians. We need to make no new assumptions other than that postulate of a point $Z=(s_o+ds)e^{i(\theta o+d\theta)}$, equation 1.1.

25.21 Secant Line Higher Order Derivatives Formulation Of Calculus

I have been trying to understand why it has been so difficult for modern theoretical physics to get it right, to make the mathematics correspond to the experimental outcomes. For example in string theory it takes two lifetimes to understand their mathematics and even then string theory gives essentially no useful results. What I have found is that there still isn't a good theoretical fit between numerical methods (e.g., used on digital computers) and the differential calculus. I also found that the solution to this problem is in simply writing dθ as a small constant let's say in the case of a smooth continuous function r(t) in Z=r+it . Recall that in differential calculus you are not allowed to write dθ in this way.

To show how this works we first recall the standard calculus method of defining a Newton quotient limit L in the case L is the derivative:

Given a number ε>0 there exists a number δ>0 such that for all t (in a given set) satisfying

$|t-t_o| <\delta$ (here $\Delta t\equiv|t-t_o|$)

we have

$|((r(t+\Delta t)-r(t))/\Delta t)-L|<\varepsilon$

Then write $\lim_{\Delta t\to 0}(r(t+\Delta t)-r(t))/\Delta t =L$, the dr/dt derivative of r in this case.

Thus you can take a smaller and smaller ε here, so then $(r(t+\Delta t)-r(t))/\Delta t$ gets closer and closer to (limit) L even if t never actually reaches t_o.

The point here is that repeatedly picking these tiny ε quantities is a "process". For example in the process of picking these ε s in doing the *local limit* of the slope for a *straight line* we find that $((r(t+\Delta t)-r(t))/\Delta t=dr/dt$ is constant (so then is dθ) but dr and dt are *not* unique and we can stop the process right there and write down our derivative-slope limit dθ. Thus **numerically** we can pick a sample *constant* dθ and say that the variation of dθ ≡δdθ=0 here (in this straight line example) but in this ε and δ choosing "process" we cannot for sure say that δdr=0 and δdt=0. On a straight line we can even self consistently pick a sample small constant ds (=|dZ|) and even then higher order derivative contributions imply dr and dt are still not unique since their contributions cancel even for this constant ds value (see section 1.1).

25.22 Point Postulate

Note a unit real single point (eg.,"1") is as mathematically *simple* as we can get but yet unit real and imaginary numbers N are equally *simple*, they just differ by the sign on the right side of $N^2=\pm1$. Also the statement that there are "No other postulates "(other than that simple "real and imaginary"(s,θ)=point Z) at least implies there is unspecified imbedding space dimensionality and coordinate axis' position (implying measurement uncertainty ds,dθ) and possibly many more such unknowns.

Occam's Razor

We start with Occam's razor.

Thus we most *simply* either postulate a real or imaginary point or in general both (ie.,complex Z). So $Z=se^{i\theta}$:

Postulate a constant **point** so $\delta Z=0$.
The osculating plane is the plane that locally a curved line resides in. A manifold is a space that can be mapped locally by a homeomorphism to R^n. Any order of derivative exists for a C^∞ manifold making it smooth and continuous with a local limit "straight line" slope of given angle θ. Also in the ε and δ limit formalism (discussed above) for a function that is a straight line the independent variable r is arbitrary (since δ can assume any value for a straight line) but slope angle θ is a constant in the oscullating (2D) plane of the curve. In this application we will have a point Z on a C^∞ manifold, thus with a straight line local limit dθ interval on this curve. Note in actuality (i.e., numerically) dθ is then a small constant with dr an ordinary (unspecified) differential in the context of that ε and δ limit formalism (i.e.,dr is always trying to be 'close', while dθ is that constant).
Thus we are on a C^∞ manifold if we can write a constant angle to a point Z as $\theta_o+d\theta$ in the oscullating plane. In that plane then $Z=(s_o+ds)e^{i(\theta o+d\theta)}$ given r, it independent are variables let's say. Note that specifying a constant (finite *or* differential) position point involves straight line coordinates (displacements) so this is how a constant position point is written *in general.*
In that regard:
1) The straight line dθ is not an added postulate because we have in our point postulate straight line coordinates even for a differential distance to a point.
2) Thus the C^∞ manifold is not an added postulate because we have a local straight line slope limit implied by dθ and thus a smooth function.
3) The two dimensional point position is not an added postulate because the osculating plane is needed to define our angle θ for the curve.
4) The requirement for polar coordinate point position is not an added postulate because a constant dθ is needed for a constant position coordinates to a point, not dr and dt because they are floating value (so not constant) differentials per above discussion of the ε, δ limit formalism.
5) The actual (i.e., numerical) designation for constant slope dθ as a small constant (because of the constant slope) is a requirement that necessarily follows from making actual measurements.
Thus you are really postulating a constant position point in a rigorous way, without making any other assumptions, by writing $Z=(s_o+ds)e^{i(\theta o+d\theta)}$. The mathematics required to do theoretical physics then follows from this postulate.

Curved Lines
As a practical matter in numerical calculations in curved space times dθ, ds become more and more *non*local as we go up in derivative. For example numerically a second derivative must be specified by 3 separate, distinct points as noted above for the second derivative of κ_{rr}.
This raises the issue of whether the calculus of higher derivatives is optimally defined to do theoretical physics. In this nonlocal context it appears that instead we need a new secant line definition for higher derivatives such
as $\lim_{h\to(M+1)K}\frac{f(x+h)-f(x)}{h}\equiv f'(x)=\frac{df(x)}{dx}$ where f(x) is the M-1 derivative, K≡one over the square root of the curvature $R=1/r^2$. Here r is the local radius of curvature of the

curve in the osculating plane. Thus we need a secant line modified definition of higher derivatives to do calculus. **We need a new formulation of calculus!**
Since h is not zero anymore for large enough M these higher order derivatives are slopes of secant lines going through points separated in the domain (M+1)K.

25.23 The Newton Quotient eq.1.5 As A Box Containment Instead of a Limit to One Side
Recall equation 1.6 is C^2 everywhere except at r=0. Also in our application the Taylor expansion 1.5a has first and second derivatives. C^2 means the function has continuous first and second derivatives (here being $1/\kappa_{rr}$) and so says that:

$$\frac{d\kappa_{rr}}{dr}=\frac{\left(\frac{(dr+r)}{dr}\right)^2-\left(\frac{dr}{dr}\right)^2}{r}$$ and

$$\frac{d^2\kappa_{rr}}{dr^2}=\frac{\frac{\left(\frac{(dr+2r)}{dr}\right)^2-\left(\frac{dr+r}{dr}\right)^2}{r}-\frac{\left(\frac{(dr+r)}{dr}\right)^2-\left(\frac{dr}{dr}\right)^2}{r}}{r}$$ exist and are continuous since r is a continuous independent variable in the bounds $0<r\leq\varepsilon<<1$ and $|dr|>0$. Note this $(0,\varepsilon)$ open interval region forms a ‘box’ and so is not a limit to one side or another as in the classical limit of the Newton quotient of elementary calculus.

The ε is the upper bound on r, with $r<\varepsilon$ and $\varepsilon<<1$. Thus we are defining a limiting region where r exists that is 'small" yet are not stating it as a 'limit' as in ordinary calculus.
We have a "small" space for r yet not a limit space. Thus equation 1.5 (and the right side $d(1/\kappa_{rr})/dr$) amounts to a *new and deeper definition of the covariant derivative*.
Thus we can now solve equation 1.5 as a differential equation in the variable r instead of as the limit of r one side of the interval. This result is further verified by the role this small r would have played in a numerical solution: the solution would still have been accurate, which is the bottom line here for doing derivatives.
This subtle difference gives awesome physics. For example on the next lower fractal scale the ε in equation 1.5 goes to sinhu as discussed in section 1.3. In chapter 2 we show that this result leads to the equation for the radius change of the universe!!!!
With the self similar Δε rotation term put in it and equation 1.9 give us our universe!!!

The point again is that this minor modification of the calculus we derived in section 1.1 is clearly *more compatible* with the most fundamental physics.

25.24 Equation 1.5 Solution r is restricted to $0<r\leq\varepsilon<<1$
Recall that the solution of equation 1.5 is given by equation 1.6: $1/\kappa_{rr}=1+\varepsilon+r_H/r$.
But flat space constant 1+ε can always be diagonalized out (i.e., made ‘1’) in $ds^2=\Sigma g_{ij}dx^idx^j$ and r_H is arbitrary and so the units of $r_Hk/rk=r'_H/r'$can then assume any form (e.g., meters, fermis,..) and any value given the arbitrary value of k.

25.25 Conformal Transformation and Covariance

Equation 1.9 has been shown to be covariant in flat space limit: that is the covariance of the ordinary Dirac equation. Also recall that equation 1.8 is used to derive equation 1.9 by simply dividing by ds so it must also be covariant. So we must show that in a coordinate transformation to curved space the form of this equation 1.8 i.e., $\alpha_x dx'+\alpha_y dy'+\alpha_z dz'=\alpha_x d\sqrt{\kappa_{xx}}dx+\alpha_y d\sqrt{\kappa_{yy}}dy+\alpha_z d\sqrt{\kappa_{zz}}dz$ (1.8)
must not change. So

1) The Dirac α matrices must keep the same form
2) and the dx must maintain the same form in such a transformation.

On point 2 we note $\sqrt{\kappa_{rr}}dr=(dr'/dr)dr=dr'$ so the form of that term is the same so point 2 is verified.

On point 1 recall from chapter 1 that the transformation to dr',dt' is conformal. In that regard note the x and y axis are rotated the same angle here with ds direction remaining a constant. Note law of cosines $H^2=x^2+y^2-|x||y|\cos\theta$ where the two vectors are in the x,y plane. Recall that θ remains 90° in a conformal transformation of orthogonal axis'. Also recall in chapter 1 we made x,y,z orthogonal in $(\alpha_x dx+\alpha_y dy+\alpha_z dz)^2=x^2+y^2+z^2$ for a flat space. Thus our conformal transformation keeps the $\cos\theta=0$ and therefore keeps the form of the orthogonality condition invariant and therefore keeps the form of the Dirac α matrices covariant. Also recall again that in the flat space case the Dirac equation has been shown to be covariant. Thus both parts of equation 1.8, the alpha α matrices and the $\sqrt{\kappa_{rr}}dr$ are covariant so equation 1.8 is covariant implying that equation 1.9 is covariant. The covariance of equation 1.9 is explicit in that chapter 5 derivation in any case.

25.26 Fractalness Implied by Postulate of Geometrical point (equation 1.1)

Recall from equation 4.14 that no observation can be made through the r_H horizon. One way of understanding the fractalness is the series given by equation 1.2. Yet another way is given again by the point postulate $\delta ds=0$, $\delta d\theta=0$. Thus again a point inside r_H is infinitesimally small so there must be a closed and bounded nested sequence of r_H intervals with the limiting interval at the point, divergence=0. Thus we have defined our infinitesimal point inside r_H in an analogous manner to how a limit is defined using epsilons and deltas in calculus. Also those 45 degree ds, dt, dr triangles (figure 21-1) on each of these r_H scales are congruent and therefore selfsimilar (fractal).

Recall also from chapter 1 that the fractalness was implied by the existence of the series elements in series 1.2
One can also write down as an alternative postulate to equation 1.1 that "there exists a smallest observable source than the observer". Recall r_H set that observability size (as in eq. 4.14). If an observer is allowed anywhere then inside r_H there must also exist an even smaller observable source (according to this postulate) and hence the fractalness is implied.

Thus there are several ways to understand the fractalness.

Conclusion

This new equation explicitly includes curved space (i.e., r_H *not* zero), thus includes force,

thus naturally explains **all** the **forces** with direct, straightforward derivation. For example at $\mathbf{r>r_H}$ the third term in the expansion of the energy term (in this new pde) gives the Lamb shift without the higher order diagrams, doesn't require the standard pathology of adding and subtracting infinities to get the QED high precission (section 17.4, ch.18).
Also at $\mathbf{r=r_H}$ it gives a bound state $2P_{3/2}$ trifolium, thus charge e spends 1/3 of its time in each lobe (fractionally charged lobes), there are 6 P states(6 flavors), the lobes can't leave (asymptotic freedom), P wave scattering (jets), explaining all the major properties of quarks (ch.3), **giving us the strong interaction without any new assumptions!!**

The standard Dirac equation on the other hand applies to **flat** space (r_H=0 there), which is a mistake to use (except for in flat space) since indeed there **are** forces.
So what do people do to try to get the experimental results after making such a egregious error?
They add in gauge after gauge, Lagrangian term after Lagrangian term, free parameter after free parameter: when their model doesn't explain new experimental results they just fudge in a new term, resulting in a big mess of a theory that confuses, stops the progress of theoretical physics dead in its tracks. Why they can prove anything this way!
In any case Dirac thought there was something missing from his equation as well, given all these pathologies resulting from trying to correct its mistakes. He advised physicists to fix his equation, which is what has been done here.

Two Steps: (1) Mathematical Structure and (2) Physics Applications (i.e., Reality)
I also wanted to point out that the postulate 1.1 is really being applied in two steps:

1) **Postulate 1 point** $Z=(s_o+ds)e^{i(\theta o+d\theta)}$ then use the **definitions** of two such points (0,0) and (1,1) to construct the positive integers and to later develop the real and imaginary numbers to work with (and so can later define the ds^2 metrics as in chapter 1 equations 1.2, 1.3) by just postulating a geometric point. *So the physics postulate also generates the underlying mathematics definitions!*
2) The next step is to use ds to define the minimal position variation of the point by collapsing the triangle inequality giving $\delta(ds)=0$, dr+dt=constant thereby giving equations 1.6-1.8. dZ/ds then gives the new pde 1.9.
   Solve it for $r<r_H$, $r=r_H$, $r>r_H$.to get the physics.

So the first step is the postulate of a point with*out* reference to its minimal variation.
The second step uses the mathematics developed in the first step to define that minimal variation

So the first step gives the mathematical underpinning, the second step gives reality.
Figuring out the nature of reality, not bad for a days work!

TERMINOLOGY

g_{ij} = Diagonalized metric component $ds^2=\sum\kappa_{ij}dx^i dx^i$ metric equation; analogous to 4 dimensional Pythagorean theorem $s^2=x^2+y^2+z^2+..$ unitless.
ds = (proper time difference)c
ψ = Dirac equation eigenfunction
$\psi\dagger\psi$ = The probability density ("lobe" 1S,2P,2S,.. structure)

ω= zitterbewegung frequency $=mc^2/\hbar$ (is zero if mass m is zero,units 1/sec).
γ_μ = Dirac gamma matrix(4X4).
V = Potential (Volts)
A_μ = Vector potential
c = vacuum speed of light $3X10^8$ m/sec
G = Newtonian gravitational constant (Nm^2/kg^2)
e = unit electric charge $1.602X10^{-19}$ C
m_e = electron mass ($9.11X10^{-31}$ kg)
∂ = partial derivative
h = Planck's constant (joules-sec)

GLOSSARY

Geometrical Representation: Physics is the procedure for observing 2D We are observing: Physics In 1 spatial dimension r with time $_t$ as the imaginary component

Thus **Postulate complex plane**: 2D invariant dZ=dr +idt and point δdZ=0

Observer Representation: Translated to Observer+Object=2⊕2=4D giving QM and new pde. New pde in QM representation With $\sqrt{g_{oo}}=\sqrt{(1-2e^2/rm_ec^2)}\equiv\sqrt{(1-r_H/r)}$ coefficient in front of old Dirac kinetic term
Strong Interaction: example: new pde $2P_{3/2}$ solutions at $r\approx r_H$
Fractal: dZ=dr+idt Has no r scale so exhibits selfsimilarity (so there are .. $>>r''_H>>r'_H>>r_H>>$..)

Heisenberg's Equation of Motion; $[H,\alpha]=i\hbar\, d\alpha/dt$ Application with new pde for **$r<r_H$, $r\approx r_H$, $r>r_H$.**

$r<r_H$ Application: Unobservability through r_H
$r<r_H$ Application: $\sqrt{(1-r_H/r)}$ Imaginary For $r<r_H$: implying r=sinhwt Expansion (Cosmology derived)
$r<r_H$ Application: Fractalness: Only r'_H and r_H New pde Hamiltonians available (Dirac Doublet Core Of the Standard Model Derived)
$r<r_H$ Application: Expansion of Zitterbewegung Cloud From r=sinhωt: (Gravity derived).
$r<r_H$ Application: Rotational Selfsimilarity With pde Spin: (CP violation derived)
$r>r_H$ Application: $r>r_H$ Application: KE+V=E Way of Writing The Energy Contribution
$r>r_H$ Application: New Potential In New pde Implies No Running Coupling Constant
$r>r_H$ Application: New Potential V in New pde Implies in new S matrix with W and Z as resonances $r>r_H$ Application: New Potential V in New pde Implies Hydrogen atom

orbital energy perturbations (Lamb Shift derived without higher order Diagrams and Renormalization)

$r \approx r_H$ Application: g_{22} gives baryon gyromagnetic ratios

$r \approx r_H$ Application: Substitute Series Ansatz For ψ In New pde: Frobenius Solution (Derive Baryon Mass Eigenvalues and Deuteron State

Consequences of Other Objects Outside M+1 r_H : (Quantized Metric with many Intriguing Implications Derived)

In Object+observer ≡ **2⊕2=4D Observer** =QM Representation (Copenhagen interpretation derived, it then is merely 4D)

Appendix A Review of Special Relativity Dirac Equation

Derivation (circa 1928)

From special relativity we get from such QM texts as Merzabacher:

$E^2=c^2p^2+m^2c^4$ (i.e, $m^2c^4=E^2-c^2p^2$) and operator form $H^2=c^2p^2+m^2c^4$ (A1)

The Dirac ansatz: $H=c\boldsymbol{\alpha}\bullet\mathbf{p}+\beta mc^2$ is used to get equation A1 (A2)

$H^2=(c\boldsymbol{\alpha}\bullet\mathbf{p}+\beta mc^2)^2=c^2p^2+m^2c^4=$ (A3)

$$(c\alpha_xp_x+c\alpha_yp_y+c\alpha_zp_z+\beta mc^2)(c\alpha_xp_x+c\alpha_yp_y+c\alpha_zp_z+\beta mc^2)=$$
$$(c\alpha_xp_x)^2+c\alpha_xp_x\alpha_yp_y+c\alpha_xp_x\alpha_zp_z+c\alpha_x\beta mc^2+$$
$$c\alpha_yp_yc\alpha_xp_x+(c\alpha_yp_yc)^2+c\alpha_yp_yc\alpha_zp_z+c\alpha_y\beta mc^2$$
$$c\alpha_zp_zc\alpha_xp_x+c\alpha_zp_zc\alpha_yp_y+(c\alpha_zpz)^2+c\alpha_z\beta mc^2 \quad \text{(A4)}$$
$$\beta mc^2c\alpha_xp_x+\beta mc^2c\alpha_yp_y+\beta mc^2c\alpha_zp_z+(\beta mc^2)^2=c^2(p_x^2+p_y^2+p_z^2)+m^2c^4$$

Therefore to return equation A1:

$\alpha_x^2=1$, $\alpha_y^2=1$, $\alpha_z^2=1$,

$\alpha_x\alpha_y+\alpha_y\alpha_x=0$, $\alpha_x\alpha_z+\alpha_z\alpha_x=0$, $\alpha_x\beta+\beta\alpha_x=0$, $\beta\alpha_x+\beta\alpha_x=0$, $\beta\alpha_y+\beta\alpha_y=0$, $\beta\alpha_z+\beta\alpha_z=0$, etc.

Recall this is the old Clifford algebra. One solution for the αs is the standard representation:

$$\alpha_x=\begin{vmatrix}0 & \sigma_x\\ \sigma_x & 0\end{vmatrix},\ \alpha_y=\begin{vmatrix}0 & \sigma_x\\ \sigma_y & 0\end{vmatrix},\ \alpha_z=\begin{vmatrix}0 & \sigma_z\\ \sigma_z & 0\end{vmatrix},\ \beta=\begin{vmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{vmatrix}$$

given that $\sigma_x=\begin{vmatrix}0&1\\1&0\end{vmatrix},\quad \sigma_y=\begin{vmatrix}0&-i\\i&0\end{vmatrix},\quad \sigma_z=\begin{vmatrix}i&0\\0&-i\end{vmatrix}$

Also note from above that $\beta\boldsymbol{\alpha}\bullet\mathbf{p}+\alpha\boldsymbol{\beta}\bullet\mathbf{p}=0$ and $(\alpha\bullet p)^2=p^2=p_x^2+p_y^2+p_z^2$.

Use these last two Clifford algebra results in the rearrangement of equation 2 as:

$H-c(\alpha\bullet p)=\beta mc^2$

Multiply by β and then define γ from

$\beta E-c\beta\boldsymbol{\alpha}\bullet\mathbf{p}=mc^2\equiv\gamma^0E-c\gamma^i\bullet p^i=mc^2$. So here, $\gamma^0\equiv\beta$, $\gamma^x\equiv\beta\alpha_x$, $\gamma^y\equiv\beta\alpha_y$, $\gamma^z\equiv\beta\alpha_z$

Use $\gamma^1p_1\psi=-i\hbar\gamma^1\partial\psi/dx^1$ =and find our Dirac equation

$\gamma_\mu\partial\psi/\partial x_\mu+i\omega\psi=0$ (A5)

Which is the original Dirac equation.

A.2 More General Derivation

Instead of starting our Dirac equation from $E^2=c^2p^2+m^2c^4$ as above we start from the more general diagonalized (since spherically symmetric) metric formulation: $ds^2=\Sigma g_{ij}dx^j dx^j = g_{xx}dx^2+g_{yy}dy^2+g_{zz}dz^2+g_{tt}dt^2$ (A6)

Note that in the Minkowski special case $ds^2=-dx^2-dy^2-dz^2+c^2dt^2$ so that after dividing by ds^2 and rearranging $c^2dt^2/ds^2=dx^2/ds^2+dy^2/ds^2+dz^2/ds^2+1$. After multiplying both sides by m^2c^4 this in fact is equation A1 $E^2=c^2p^2+m^2c^4$

In any case analogously again divide both sides of equation A6 by ds^2 to obtain $1=g_{xx}dx^2/ds^2+g_{yy}dy^2/ds^2+g_{zz}dz^2/ds^2+g_{tt}dt^2/ds^2$

Then define the 4X4 α_i matrix from $(\alpha_i\bullet\sqrt{g_{ii}}dx^i/ds)^2=g_{xx}dx^2/ds^2+g_{yy}dy^2/ds^2+g_{zz}dz^2/ds^2$ (Analogous to the above $(\alpha\bullet p)^2=p^2=p_x^2+p_y^2+p_z^2$)

After plugging in α_i we get $(c\alpha_x\sqrt{g_{ii}}dx^i/ds+c\alpha_y g_{yy}\sqrt{g_{22}}dx^2/ds+c\alpha_z\sqrt{g_{33}}dx^3/ds+\beta mc^2\sqrt{g_{oo}}dx^o/ds)=$

Set $p_x=\sqrt{g_{ii}}dx^i/ds$, $p_y=\sqrt{g_{22}}dx^2/ds$, $p_z=\sqrt{g_{33}}dx^3/ds$, $E=\sqrt{g_{oo}}dx^o/ds$. (A7)

We iterate just in the above equation A4

$(c\alpha_xp_x+c\alpha_yp_y+c\alpha_zp_z+\beta mc^2)(c\alpha_xp_x+c\alpha_yp_y+c\alpha_zp_z+\beta mc^2)=$
$(c\alpha_xp_x)^2+c\alpha_xp_x\alpha_yp_y+c\alpha_xp_x\alpha_zp_z+c\alpha_x\beta mc^2+c\alpha_yp_yc\alpha_xp_x+(c\alpha_yp_yc)^2+c\alpha_yp_yc\alpha_zp_z+c\alpha_y\beta mc^2=^- c\alpha_zp_zc\alpha_xp_x+c\alpha_zp_zc\alpha_yp_y+(c\alpha_zp^z)^2+c\alpha_z\beta mc^2$ (A8)
$\beta mc^2c\alpha_xp_x+\beta mc^2c\alpha_yp_y+\beta mc^2c\alpha_zp_z+(\beta mc^2)^2=c^2(p_x^2+p_y^2+p_z^2)+m^2c^4$

Therefore to return equation A1:

$\alpha_x^2=1$, $\alpha_y^2=1$, $\alpha_z^2=1$, $\alpha_x\alpha_y+\alpha_y\alpha_x=0$, $\alpha_x\alpha_z+\alpha_z\alpha_x=0$, $\alpha_x\beta+\beta\alpha_x=0, \beta\alpha_x+\beta\alpha_x=0$, $\beta\alpha_y+\beta\alpha_y=0$, $\beta\alpha_z+\beta\alpha_z=0$, etc. Recall this is the old Clifford algebra. Multiply by β as in $\gamma^o\equiv\beta$, $\gamma^x\equiv\beta\alpha_x$, $\gamma^y\equiv\beta\alpha_y$, $\gamma^z\equiv\beta\alpha_z$. Substituting back in $p_x=\sqrt{g_{ii}}dx^i/ds$, $p_y=\sqrt{g_{22}}dx^2/ds$, $p_z=\sqrt{g_{33}}dx^3/ds$, $E=\sqrt{g_{oo}}dx^o/ds$. And after multiplying by ψ and setting as usual in the operator formalism $\sqrt{g_{11}}\gamma^1p^1\psi=-ih\sqrt{g_{11}}\gamma^1\partial\psi/dx^2$ we have finally that $\sqrt{g_{11}}\gamma^1\partial\psi/dx^1+\sqrt{g_{22}}\gamma^2\partial\psi/dx^2+\sqrt{g_{33}}\gamma^3\partial\psi/dx^3+\sqrt{g_{oo}}\gamma^o\partial\psi/dx^o+i\omega\psi=0$
$=\sqrt{g_{\mu\mu}}\gamma^\mu\partial\psi/\partial x^\mu+i\omega\psi=0$ (A9)

A.3 Relativistic Covariance

Also we construct the Relativistic invariance (as in Merzbacher, Quantum Mechanics 2ed, p.576). In that regard we construct the coordinate transformation $x'^\mu=a^\mu{}_\nu x^\nu+b^\mu$ with real coefficients $a^\mu{}_\nu$ and b^μ subject to the orthogonality condition

$ds^2=\Sigma g_{\mu\nu}dx^\mu dx^\nu = dx'^\mu dx'_\mu=dx^\nu dx_\nu$ or $a^\mu{}_\lambda a^\nu{}_\mu=\delta^\nu{}_\lambda$ (A10)

For each such geometrical transformation there must exist a spinor transformation

$S=\exp^{[(-i/2)\theta\, \hat{n}\, \Sigma]}$ (A11)

such that a unitary matrix U takes a state vector Ψ into another state vector Ψ'according to $U^t\psi(r',t')U=S\psi(r,t)$. It is well known that the equation A10 Dirac equation implies a equation A11 geometrical spinor transformation S and so proving the relativistic invariance of the ordinary Dirac equation A10. Next we replace the equation 5.1 Minkowski $g_{11}=-1$, $g_{22}=-1$, $g_{33}=-1$, $g_{oo}=1$ with a general relativistically covariant form g_{11}, g_{22}, g_{33}, g_{oo} in equation 1. Then equation 5.1becomes instead after merely carrying through the $g_{\mu\nu}$ in the above derivation (i.e., instead of the $g_{\mu\mu}$= -1,-1,-1,1):

$\sqrt{g_{\mu\mu}}\gamma^\mu\partial\psi/\partial x_\mu+i\omega\psi=0$ (no sum on μ) (A5) which is the subject of the paper. In that regard we could also define a new Dirac gamma matrix called γ'^{μ} equivalently to how γ^{μ} is defined in equation A9:

$$(\Sigma_{\mu}\gamma'^{\mu}p_{\mu})^2 \equiv (\Sigma_{\mu}g_{\mu\mu}p^{\mu 2}) \quad \text{(A12)}$$

and later linearize with spinor ψ as in chapter 5.3. This converts equation 9 into the old equation A5 form

$$\gamma^{\mu}\partial\psi/\partial x^{\mu}+i\omega\psi=0 \quad \text{(A13)}$$

QED

Also the above equation 5.7 covariance can also be shown by substituting: $\left(\sqrt{g_{ii}}\,p^i\right)^2 \equiv \left(\gamma'^j\,\nabla_j\right)^2$ in the linearization step.

In any case the relativistic covariance of equation 5.5 is guaranteed because it was derived from $ds^2=\Sigma g_{ij}dx^i dx^j$

On the other hand if you use these algebraic definitions of the integers then Godel has shown an incompleteness in mathematics. Thus (given the results of chapter 7 again) here we use the Peano formulation (not as axioms anymore) and avoid Godel's incompleteness problem altogether.

This theory is then complete

Appendic B 2 Dimensionality Simplifications

B.1 The String Theory 2D dA Must Become $dA = ids_t ds_r$ With No Higher Dimensionality Allowed (at least they tried 2D, then botched it with the 11D)

That postulated invariant path dZ=dri+idt (recall $ds^2 \equiv \sum\kappa_{ij}dx^i dx^j$) and which has a new $\sqrt{\kappa_{oo}}$ coefficient ($\kappa_{oo}=1-2e^2/rm_ec^2$) replaces that generic $d\sigma$ in string theory with $ds_t ds_\rho=dA$ in that 2D string action. Recall integrated invariant paths $S = \int ds$ are equivalent to actions and so with a mathematics that can be made equivalent to the Lagrangian-Hamilton's principle formulation. Out of this string theory will then come a single Hamiltonian H_o that is equivalent to our equation 1.9. This Hamiltionian should not be meddled with by trying to immerse it in 11 dimensions. Thus, as in chapter 1, we add the 2D observer and 2D object then to give our $2\oplus 2=4$ direct sum degrees of freedom for the observer-object system: our physical system. So string theory needed to stop at 2D and not get all bollixed up by the mathematics of higher dimensions.

B.2 In Contrast Successful Theories Show The Advantage of 2D

Related ideas in standard literature imply a **lower dimensionality** as well. Recall from chapter 1 that we have a **2D theory here**: we postulated a 2D coordinate point dZ. Equation 1.9 is explicitly 4D (x, y, z, t) but the constraints of equations 1.5 and 1.6 still makes the theory a 2 degree of freedom theory, 2D, in analogy with how the Bianchi identities reduce the degrees of freedom of the Einstein equations from 10 to 6. Note in standard QM these constraints are completely ignored (e.g., there are no nontrivial κ_{ij} in the ordinary Dirac equation: $r_H=0$ there), hence the above 4D Copenhagen interpretation implications. From arXiv:hep-th/0203101: "More fundamentally, t'Hooft and Susskind used the laws of black hole thermodynamics to argue for a general Holographic Principle of nature, which asserts that consistent theories of gravity and quantum mechanics **must be lower dimensional**. Though not yet fully understood in general, the holographic principle has led to the *only complete theories of quantum gravity*, such as the AdS/CFT correspondence."In that regard it is *un*fortunate that string theory did not stick to its original 2D string (i.e., thus including time). *They would have hit the jackpot then* (i.e, eq.1.9). Also see section 25-8. Instead they immersed their string in that 11 Dimensional quagmire.

BIBLIOGRAPHY

1. A.Einstein, *The Principle of Relativity*,(Dover) pp.119
2. H.L.Royden, *Real Analysis*, 2nd Ed., pp.127
3. L.P.Eisenhart, *Riemannian Geometry,* (Princeton University Press), Section 28.
4. S.Hawking, *Large Scale Structure of SpaceTime*, pp.200.
5. H.Minkowski, *Space and Time*, pp.87
6. Sokolnikoff, *Tensor Analysis*, pp.304
7. S.Hawking, *Large Scale Structure of Space Time*, pp.309...instead he will see O's watch apparently slow down and **asymptotically** (during collapse) **approach** 1 o'clock...", So g_{11} in practical terms never quite becomes singular and so ε approaches but never really reaches 1. Yet the external observor sees very rapid rebound (note zitterbewegung discussion below) near $r=r_{bb}$, corresponding to $\varepsilon=1$ so "r" never actually is r_{bb} .
8. Kerr.R. Phys.Rev.Letters, **11**, 237 (1963)
9. S. Weinberg, *Gravitation and Cosmology*, pp.179
10. S.W.Hawking, *Large Scale Structure of Space Time*, pp.369
11. S.Weinberg, *Gravitation and Cosmology*, pp.394
12. Max Tegmark 1997,arXiv:quant-ph/97/09032v1. 15 Sept 1997
13. J.S.Bell, *Speakable and Unspeakable in Quantum Mechanics*, 1987, Cambridge, pp.84
14. Maker, David, "*Quantum Mechanics and Fractal Space-Time*", Solitons, Chaos, Fractals, Jan10, 1999
15. Sokolnikof, *Tensor Analysis*, 2nd Ed,Wiley, 1964, equation 105.9
16. S.Hawking, *Large Scale Structure of SpaceTime*, pp.372
17. S.Hawking, *Large Scale Structure of SpaceTime*, pp.309.
18. Maker,David, *Oregon Graduate Institute*, MISC-1, 1995, equation 30
19. J. Bjorken, *Relativistic Quantum Mechanics*, McGraw Hill, 1964, p.84
20. Halzen F, A.D.Martin, Quarks and Leptons, Wiley, 1984
21. E.Merzbacher, *Quantum Mechanics*, p.509
22. ME Szabo. The Collected Works of Gerhard Gentzen. North-Holland Publishing Company, 1969
23. S. Weinberg, *Gravitation and Cosmology*, p.392
24. R. Kerr, Phys. Rev. Letters,11,237(1963)
25. Liboff, Richard,*Quantum Mechanics*, 2nd ed.,1991, p.202
26. Guidry, *Gauge Fields*, P.236
27. H.B.G.Casimir, Phys.Rev. 73(1948)
28. Mordehai Milgrom, *Astrophysical Journal*, Vol.270, p365-370, July 15,1983

29. El Naschie, M.S., Nottale,L.Al Athel,S and Ord,G.,*Fractal Spacetime and Cantorian Geometry in Quantum Mechanics*, Chaos Solitons & Fractals, June 1996, 7(6)
30. M.Agop, G.Ciobanu,*Cantorian $\xi^{(\infty)}$ space-time in Cartan, de Broglie and field theories*, Chaos, Solitons and Fractals, Volume 14, Number 6, Oct. 2002
31. Faddeev, Slavnov, Gauge Fields,..Benjamin Cummings Publishing, 1981.
32. Enge, Harold, Introduction To Nuclear Physics, 1966, Addison Wesley, page. 45
33. Misner, Thorne, *Gravitation*, 1973