

# Part IV

## 4D *IS* Quantum Mechanics

### Chapter 21

#### 21.1 QM is The Result of 4 Dimensionality

Review: Copenhagen interpretation is merely 4D

Recall in chapter 1 section 1.3 that Object+observer  $\equiv 2\oplus 2=4D=QM$  Representation (Copenhagen Interpretation Derived)

Observer 2D. Thus add 2D observer and get 4Degrees of freedom and the new (dirac) pde.  $(\sum_{\mu} \gamma^{\mu} \partial \psi / \partial x^{\mu} - \omega \psi = 0)$  with its  $\psi$ .\*

Thus in adding the 2D observer you have collapsed the wave function to the observed  $\psi$ : hence the Copenhagen interpretation(CI) of QM is essentially a 4D theory.

Recall we postulated a 2D unambiguous  $dZ=dr+idt$ . in the geometrical representation. We introduce the observer's 2D as well and obtain  $2\oplus 2=4$ Degrees of freedom and that new pde with its  $\psi$  and boundary conditions in the observer (or QM) translation. Also this postulate states no (certain)  $r$  or  $p$  and so allows the uncertainty principle and does not allow hidden variables because this postulate is fundamental. Thus the introduction of the observer results in 4D and the collapse of the wavefunction to  $\psi$ . Thus the **Copenhagen interpretation of quantum mechanics is really a statement of 4D.**

#### 21.2 Contrast with Other QM Interpretations

We begin by defining:

$$U = \text{time development operator} = e^{-iHt/\hbar}$$

$\uparrow$  = spin up

$O_i$  = initial observer state

$O_{uf}$  = state of observer for spin up after observation

For the moment we return to the nonrelativistic approximation to the Dirac equation, the Schrodinger equation representation. Thus in that representation, if an atom is originally in a superposition of  $\alpha(\uparrow) + \beta(\downarrow)$ , then the state resulting from the observer interaction is:

$$U((\alpha(\uparrow) + \beta(\downarrow)) \otimes |O_i\rangle) = \alpha|\uparrow\rangle \otimes |O_{uf}\rangle + \beta|\downarrow\rangle \otimes |O_{df}\rangle$$

If  $\alpha|\uparrow\rangle \otimes |O_{uf}\rangle$  is what is then measured then in the (dominant) Copenhagen interpretation  $\beta|\downarrow\rangle \otimes |O_{df}\rangle$  is just (arbitrarily) dropped and we say that the wave function has collapsed to the final measured value  $\alpha|\uparrow\rangle \otimes |O_{uf}\rangle$ . Unfortunately in this interpretation this postulated wavefunction collapse by definition happens instantaneously over all space so implies nonlocality and superluminal communication. In many worlds (MWI) on the other hand we don't drop the second term and simply ignore it (call it the second world of many such possible worlds if you want, (Tegmark 1997) even though the second state adds energy to the system and so can lead to a violation of the conservation of energy. But we still do avoid those wave function collapse conundrums with MWI. In Bohm's interpretation we add adhoc hidden variables to the system along with other adhoc complications. Thus all three QM interpretations appear either adhoc or incorrect. There is a 4<sup>th</sup> way with no such liabilities that is discussed extensively in the first part of this book. Just note that we can construct the Copenhagen interpretation by bringing in the 2D observer and therefore the collapse to  $\psi$ . The Copenhagen interpretation just becomes

4D, a simple, elegant understanding of what otherwise, over the past 70 years at least, has been taken to be a pretty mysterious concept. Furthermore the Schrodinger equation is nearly parabolic and so boundary conditions appear to apply (nearly) instantaneously whereas the Dirac equation is near hyperbolic and the path integral gives light speed limited information propagation. Since the Schrodinger equation is a special low energy case of the Dirac equation then velocity of information transmission is still limited to below  $c$  propagation by the Dirac equation. Thus entangled states do not give information moving faster than  $c$ . This won't let us get away with claiming that superluminal communication is a result of wavefunction collapse created by bringing in the observer to create the 4D. Also we needn't postulate wave function collapse since bringing in the observer gives  $2+2=4$  degrees of freedom which are then restrained by saying the Schrodinger boundary conditions are responsible and thereafter constraining them by the Dirac equation.

### Origin of Quantum Mechanics

Recall point postulate equation 1.1 of  $Z=(s_0+ds)e^{i(\theta_0+d\theta)}$ .

Given it is a constant point  $\delta d\theta=0$  and  $\delta ds=0$ . Since  $ds^2=dr^2+c^2dt^2= dr^2+dt^2$ .  $c$  is taken to be a constant here and we have thereby derived special relativity.  $\delta d\theta=0$  is a constant phase condition that then gives the quantum mechanics because the slope  $dr/cdt=$  constant and so  $k(r-ct)=0$ . We can add this "0" to  $d\theta$  and take the derivative of  $Z$  with respect to  $t$  and get  $ikc$  and then define the coefficient  $ikc$  as the momentum and we have then the operator formulation of quantum mechanics.

Recall from section 1.1 that we look at higher derivatives  $\partial^M(\delta d\theta)/\partial\theta^M$  giving nonlocal (non point) effects and an added (observer) 2 degrees of freedom  $dr', dt'$  and resulting generally covariant generalization of the Dirac equation that does not require gauges, that new pde equation 1.9. The new pde explicitly then is not give the  $\psi$  without the observer and we will have derived the Copenhagen interpretation of quantum mechanics.

**Klein paradox:** does not occur with equation 1.9, that generalized Dirac equation. There are always reflection eigenfunction solutions at very high potential barriers that are smaller amplitude than the input eigenfunctions which is not true for the old Dirac equation.

### 21.3 Covariant Gaugeless Quantization of the Electromagnetic Field

The standard quantization of the electromagnetic field treats the vector potential very differently from the scalar potential so the standard method is not covariant. Also, that method requires the Coulomb gauge 3D divergence and gives a sum of plane wave solutions at the end and thus there is no hint of the pulse nature, lets say of a gamma ray photon (click on a Geiger counter lets say) even though we obtain the second quantization creation and annihilation operators from this method.

Clearly something is wrong if there is not covariance and **from my own work** something is also wrong if you even require a gauge (Section 12.3). Note for spin 1 photon with left and right helicity  $[a_R(k), a_R^t(k')] = [a_L(k)] = [a_L(k), a_L^t(k')] = \delta_{kk'}$ .

Recall you need the commutation relations for the R,L helicity creation-annihilation operators to give the Bose Einstein statistics associated with the photon. The reason for doing the quantization of the electromagnetic field is to derive creation and annihilation operators of individual photons (analogous to the creation annihilation operators for the Dirac particles) in

$$H = \sum \hbar \omega_k (a_R^t(k) a_R(k) + a_L^t(k) a_L(k)).$$

Our method for introducing covariance is to set the D'Alembertian = 0 requiring that we add a scalar potential term to the standard vector potential (Merzbacher QM, page 556, 2<sup>nd</sup> ed. Wiley) quantization term; i.e., sum to 4 *instead* of 3 in:

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k) e_k^{(1)} + a_2(k) e_k^{(2)}] e^{ikr}$$

Also given the Heisenberg equations of motion (chapter 10)

$$i\hbar \frac{da(k,t)}{dt} = [a(k,t), H] \quad (21.1)$$

we create a new factor  $e^{i(kr-\omega t)}$  in the potential.

The 4 vector quantization of the electromagnetic field

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k) e_k^{(1)} + a_2(k) e_k^{(2)}] e^{ikr}$$

then goes to the time dependent form

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k) e_k^{(1)} + a_2(k) e_k^{(2)}] e^{ikr-i\omega t} \quad (21.2)$$

where the usual 3 vector potential components are given by

$$A^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^3 \frac{1}{\sqrt{2\omega_k}} [a_1(k) e_k^{(1)} + a_2(k) e_k^{(2)}] e^{ikr-i\omega t}$$

with the scalar component is given by

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{\sqrt{2\omega_k} L^{3/2}} [a_1(k) e_4^{(1)} + a_2(k) e_4^{(2)}] e^{ikr-i\omega t}$$

In the vector potentials the creation annihilation operator vectors can be rotated, since they are first rank tensors, so the polarization direction is perpendicular to the k vector as is required for the spin 1 E&M field. Thus we have new operators  $a_1^t(k) \cos \theta + a_1^t(k) \sin \theta$  and

$-a_1^t(k) \sin \theta + a_1^t(k) \cos \theta$ . The coefficients in the scalar potential and the angle  $\theta$  are chosen (also as if originating in a 3D a rotation) so that the D'Alembertian box, instead of the divergence, dotted into the four potential is zero. Since scalar V can be in general be time and space dependent this *angle can be dynamic*, changing in both space and time (x,t) to keep the D'Alembertian equal to zero. In any case you get the covariant Lorentz gauge condition instead of the noncovariant Coulomb gauge condition where the  $e_4^1$  and  $e_4^2$  are defined so that, along with the usual transverse polarization vectors (to the k vector), the orthogonality of the 4 vectors  $e_k$  and k results in

$$\square \bullet \mathbf{V}_\mu = \nabla \bullet \mathbf{A}(r,t) - \partial V / \partial t = 0 \quad (21.3)$$

The  $e^{ikr-i\omega t}$  term implies a wave equation given by

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) V = 0, \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{A} = 0 \quad (21.4)$$

Which appears to be the old Lorentz gauge formulation of the Maxwell equations. In that regard note as usual  $B = \nabla \times A$ , and  $E = -\partial A / \partial t$  (Faraday's Law) using the A. This result is put into  $H = \sum \hbar \omega_k (a_R^t(k) a_R(k) + a_L^t(k) a_L^t)$  converting it into an integral over  $E^2$  in the usual way.

So why is 21.3 and 21.4 gaugeless as well? In that regard I found a generally covariant generalization of the Dirac equation that does not require gauges (<http://davidmaker.com>). The  $\kappa_r \kappa_{o0} = 1$  equation (eq.1.5), after taking derivatives and using the Heisenberg equations of motion, becomes the Lorentz gauge form of the Maxwell equations. This result then forces those  $\theta$  angles in that rotation of the  $a_{1l}$  creation operators to be dynamic, changing with time as we discussed earlier.

Note also that these Maxwell equations use four potentials instead of E and B fields even though you can still form the E and B fields in the usual way ( $\text{curl} A = B$ ,  $dV/dr = E$ ). Thus in this theory the Lorentz gauge is not a gauge anymore so indeed *we do have a covariant, gaugeless quantization of the E&M field*. As you recall Ahronov-Bohm implies such a potential, as opposed to field dependence of physical reality. Note that 21.3 and 21.4 is the Lorentz gauge explicitly covariant form of the Maxwell equations (section 16.2). But 16.2 are the actual physical (Maxwell) equations in potential form (special case of electroweak field equations) and so are not gauged here (section 12.3). Thus, this is a gaugeless and covariant quantization of the electromagnetic field. In any case you get the covariant Lorentz gauge condition instead of the noncovariant Coulomb gauge condition and in the end obtain the second quantization condition for the electromagnetic field:  $H = \sum \hbar \omega_k (a_R^t(k) a_R(k) + a_L^t(k) a_L^t)$ .

### 21.4 Beat Frequency Envelope and the Uncertainty Principle

Just because you find creation and annihilation operators for the E&M field does not mean you have not localized a photon. Equation 21.2 gives plane wave sums of energy  $\hbar \omega$  with no hint, for example, that the sum is the pulse of a gamma ray click on a Geiger counter. It could be anything. So, how then can plane waves ever give rise to local clumps of energy (called 'photons' in the in the Maxwell equation case)? In that regard, recall that the probability density interpretation of  $\psi^* \psi$  motivates the uncertainty principle. But the uncertainty principle in time and hence in frequency) for multiple emitters allows waves (here the plane wave solutions to Maxwell equations) of differing frequency  $\Delta \omega = \frac{1}{2}(\omega - \omega')$  in a plane wave to form "beats" of the plane wave. Hence the origin of photons, here as merely these beats!!!! The prosthaphaeresis formula  $\cos \omega' t + \cos \omega t = 2 \cos(1/2(\omega' + \omega)t) \cos(1/2(\omega - \omega')t)$  then gives this beat frequency as  $\Delta \omega$ .

Next we show that the energy of one of these uncertainty principle beat frequency envelopes is  $E = \hbar \omega$  for the SHM and for the resulting E fields this oscillation creates (as in an antenna). Note these are SHM  $\psi$ s so we can then use the SHM Schrodinger equation Hamiltonian expectation values. The E&M electric fields also must have the same amplitude dependence and the consequent energy transitions must also be the same as the SHM creating them thus constraining integration limits and amplitudes. To find the energy of a single beat frequency envelope given by this use of the uncertainty principle we can then substitute the  $\psi$  into the Schrodinger equation expectation value of the Hamiltonian  $E = \int \psi^* (-i \hbar d/dt) \psi d\tau$ . We correct for the cosine not being the shape of a minimal uncertainty wave packet (with the .2067 integration limit below) and take the integrations symmetrically about the center so the  $\cos \omega t$  (for average  $\omega$ ) comes out of the integral. We normalize a one dimensional  $\psi$  over beat length L which is the minimal  $\Delta x$  uncertainty corresponding to the above  $\Delta \omega$  ( $n=2$  for  $\Delta x = 2L$ ).

$$\frac{\int_0^{G\Delta\omega\Delta t/2(\max)} f(\omega)\cos(\Delta\omega\Delta t/2)d(\Delta\omega\Delta t/2)}{\Delta\omega\Delta t/2[\max]} = \frac{\int_0^{.2067} f(\omega)\cos\theta d\theta}{.25} = 1.0$$

by setting the integration limit equal to .2067 and  $f(\omega) = 1$

$$E = \int \psi^* (-i\hbar/dt)\psi d\tau = \int [\cos(\varpi t)] \cos(\Delta\omega\Delta t)/G\sqrt{\Delta x}^* (-i\hbar/dt) [\cos(\omega t)] \cos(\Delta\omega\Delta t)/G\sqrt{\Delta x} d\tau =$$

$$-i \int_{-\Delta x/2}^{\Delta x/2} \left[ \frac{\cos(\varpi t)}{\sqrt{\Delta x}} \left( \frac{\int_0^{G\Delta\omega\Delta t/2(\max)} \cos(\Delta\omega\Delta t/2)d(\Delta\omega\Delta t/2)}{\Delta\omega\Delta t/2[\max]} \right) \frac{d}{dt} \left( \frac{\cos(\omega t)}{\sqrt{\Delta x}} \left( \frac{\int_0^{G\Delta\omega\Delta t/2(\max)} \cos(\Delta\omega\Delta t/2)d(\Delta\omega\Delta t/2)}{\Delta\omega\Delta t/2[\max]} \right) \right) \right] dx$$

$$-i \left( \int_{-\Delta x/2}^{\Delta x/2} \text{real} \left( \frac{e^{-i\varpi t}}{\sqrt{\Delta x}} \right) (1) \frac{d}{dt} \left( \text{real} \frac{e^{i\omega t}}{\sqrt{\Delta x}} (1) \right) dx \right) i =$$

Taking in general imaginary sine and real cosine components of the Schrodinger equation plane wave and multiplying both sides by  $\hbar$  to set the energy units:

$$- \hbar i \left( \int_{-\Delta x/2}^{\Delta x/2} \left( \frac{e^{-i\varpi t}}{\sqrt{\Delta x}} \right) \frac{d}{dt} \left( \frac{e^{i\omega t}}{\sqrt{\Delta x}} \right) dx \right) i = -i \left( \varpi \int_{-\Delta x/2}^{\Delta x/2} dx / \Delta x \right) i = E = \hbar\varpi$$

which is the energy of the photon. You might even say that the uncertainty principle (when applied to a plane wave) then demands the existence of photons and the other particles.

Thus, in contrast to the usual 1) lack of covariant quantization, 2) Coulomb gauge E&M field quantization, 3) lack of specificity on the pulse nature of the photon, 4) many and varied interpretations of QM, etc.,

- 1) Here the Quantization of the E&M field is Covariant
- 2) The Quantization of the E&M field doesn't require a gauge
- 3) Here we have the pulse nature of the photon as well (e.g., gamma ray click on a Geiger counter)
- 4) The simplest way to understand (Copenhagen interpretation) quantum mechanics is that it is 4D, the mystery is solved.

## 21.5 More Consequences of Reducing the Dimensionality: Possible Origin of the Uncertainty Principle

That standard proof of the uncertainty principle is 1) Bohr postulate of  $\psi^*\psi$  as the probability density and then 2) using the Cauchy Schwarz inequality on the integral of the expectation value of  $(x-\Delta x)(p-\Delta p)$ . This has the appearance of proving what you are assuming since the Bohr postulate implicitly contains this uncertainty idea. Also the Bohr postulate seems like an extra postulate added on to our equation 1.1 postulate of a geometrical point.

But perhaps there is a deeper meaning to the uncertainty principle, one in which that integral provides a uncertainty  $\Delta x$  region analogous to the Rayleigh region in optics, hinted at by the fact that both are wavelength dependent effects. In that regard why can't truth (i.e., psf position and intensity information) be extracted from deep within the Rayleigh region even for high SNR?

The answer to the above question is that we must "reduce the dimensionality" of the problem to extract this information by finding ridges (in intensity space) using simple slope thresholds along circles around the intensity maximum. These ridges then form lines, which are *one* dimension instead of the 2D image. Thereafter we initialize our least squares search on these lines. This is the solution to the problem of super-resolution. So we never test on the ghost images making this method hit the Cramer Roa lower bound dead on. I did my own experiments with real optical systems to verify this method worked (2007 mdsea paper, my method is called DRCSO).

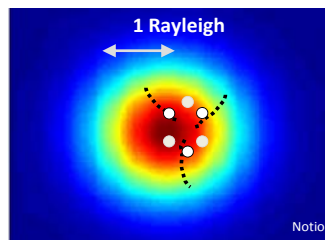
In any case within the diffraction limit those "ghost" images, or artifacts, seem to be connected to the truth configurations via simple  $SO(2)$  or translational group symmetries. The simpler the group symmetry here the smaller the least squares differences suggesting a group theoretical configurational origin of that subrayleigh image ambiguity problem.

The reason for mentioning this configuration ambiguity problem here is that it may then be the underlying source of the Bohr-Born postulated probability density  $\psi^*\psi$  (so implied uncertainty) which here then is seen to originate in that CSO position ambiguity (remedied by DRCSO) so it then is a derived result, not postulated. Therefore *we still have only ONE postulate* eq. 1.1, Chapter 1.

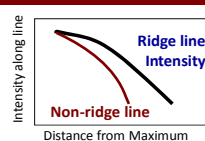
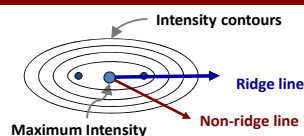
### The Dimensional Reduction Set

**Maker's observation:** Source centers *tend* to lie along ridge lines that have negative curvature in all directions—Saddle lines excluded.

- The DR Set is a sampling of points along ridge lines
- Initialize optimization by selecting points from the DR Set
- There must be enough Signal-to-Noise to find the ridge lines



**Ridge lines of interest are lines of slowest monotonic descent**



## 21.5 Summary of Theory

Postulate (Occam's razor) constant position  $\delta Z=0$  point in this most general way

$$Z=(s_0+ds)e^{i(\theta_0+d\theta)} \quad 1$$

( $=r+it$ ) leads to several equations:

Explicitly  $\delta ds=0$  and also  $\delta d\theta=0$  implies  $\delta(dr+dt)=0$  (fig.21-1) 2

Thus the  $dr+dt=\text{constant}=dr'-\varepsilon/2+dt'+\varepsilon/2$ , Solve for  $dr/dr'=1+\varepsilon-r_H/r$ .

So  $\delta(dr+dt)=0$  is a second order differential equation with two integration constants:  $\varepsilon$  and  $r_H$ .

We write  $dZ/ds$  in terms of these two integration constants to connect to observables.

$dZ/ds$  then has that **new pde** structure. 3

Writing  $dr/dr'$  in terms of these two integration constants and noting the  $C^\infty$  implications of  $\delta d\theta=0$  gives us general relativity.

Displacement  $\varepsilon$  allows new points (besides the postulated point) and so new  $r_H$  integration constants.

Thus using set builder notation we can define Mth fractal scales

$F_M=\{x \in \mathbb{R} \mid r'_H < r_H, x = \varepsilon < r_H\}$ ,  $F_{M-1}=\{x \in \mathbb{R} \mid r''_H < r'_H, x = \varepsilon < r'_H\}$ ..etc . The  $r_{Hs}$  are 'horizons' and there are observability issues through horizons.

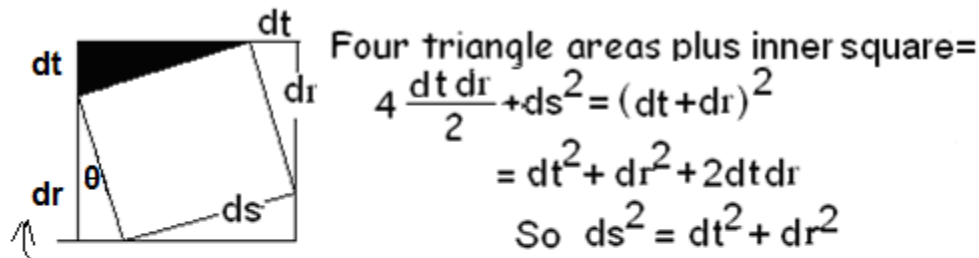
So we solve our equation 3 (new pde) inside ( $x < r_H$ ) each such horizon.

Solving that new pde for the resulting interior ( $r < r_H$ ) cosmological expansion gives  $r=r_0 \sinh \omega t$ .

Anyway, there is no need for free parameters, gauges, with this approach and you get the physics (see Ch.1, <http://davidmaker.com>).

**Explanation of Figure 21-1:  $dr+dt=\text{constant}$  and  $\delta(ds)=0$  Implies  $|dr|=|dt|$**

Recall section 1.1.



**Here  $dr+dt=\text{constant}$ . Within this constraint  $ds$  is smallest, thus an extrema when also  $dt=dr$ ,  $\theta=45^\circ$  as in equation 1.2a..**

**Also note that  $dr+dt=\text{constant} = ds/\sqrt{2}$  at  $\theta=45^\circ$ .**

**The hypotenuse defines the invariant  $ds$  in equation 1.3.**

**Figure 21-1**

Note when  $dr+dt=\text{constant}$  and  $\delta(ds)=0$  and so  $\text{constant}=ds\sqrt{2}$  and  $dt=dr$ .

### Short Summary

Here is that summary of the physics:

Postulate constant position ( $\delta Z=0$ ) geometrical point: **Z**.

Note that one result of this simple postulate is a second order differential equation.

Plug its two integration constants ( $\varepsilon$  and  $r_H$ ) into  $dZ/ds$  to get that new pde, eq.1.9

$\varepsilon$  is a coordinate translation so we have more points and new  $r_H$  horizons, thus a fractal space time.

There can be no more rigorous or fundamental ways of doing theoretical physics!

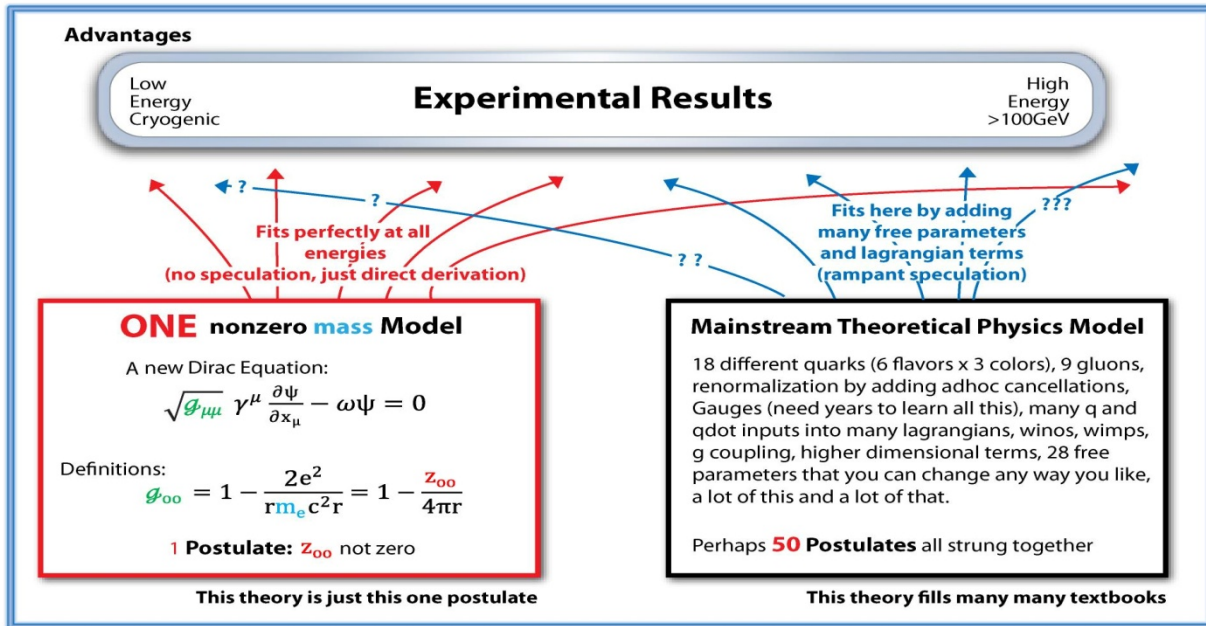


Figure 21-2

**References**

Bell J.S., *Speakable and Unspeakable in Quantum Mechanics*, 1987, Cambridge, pp.84  
 Max Tegmark 1997, arXiv:quant-ph/97/09032v1 15Sept 1997